Notational conventions for elementary geometry

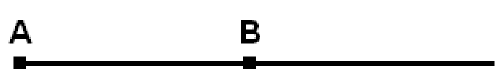
Symbolic notation will make it easier to formulate many statements, but unfortunately there are no well – established notational conventions for many key concepts. Therefore we have summarized the main features of our notation for reference purposes. We begin with eight conventions involving lines. Many are surely self – explanatory, but a few might seem arbitrary.

1. Given two points **A** and **B,** the unique ***line*** joining them will be denoted by **AB.**
2. Given two points **A** and **B,** the ***closed line segment*** joining them, which consists of **A, B,** and all points **X** which lie between **A** and **B,** will be denoted by **[AB];** similarly, the ***open line segment*** joining them, which consists of all points **X** which lie between **A** and **B,** will be denoted by **(AB).**
3. Given two points **A** and **B,** the ***closed ray starting at* A *and passing through* B,** which consists of **A, B,** all points **X** which lie between **A** and **B,** and all points **X** such that **B** lies between **A** and **X,** will be denoted by **[AB.** Equivalently, this is the set of all points **X** on the line **AB** such that **A** is **NOT** between **X** and **B.**
4. Similarly, the ***open ray starting at* A *and passing through* B,** which consists of **B,** all points **X** which lie between **A** and **B,** and all points **X** such that **B** lies between **A** and **X,** will be denoted by **(AB.**
5. One also has the ***half open intervals*** **[AB)** and **(AB]** which contain one endpoint (indicated by a square bracket) but not the other (indicated by a round parenthesis).
6. The statement that ***a point* X *lies between* A *and* B** will often by written symbolically as **A∗X∗B.**
7. Given two points **A** and **B,** the ***distance*** from **A** to **B,** or equivalently the ***length of the closed segment*** **[AB],** will be denoted by **|AB|.**
8. Given three noncollinear points **A, B** and **C,** the ***triangle*** with vertices **A, B, C** — written △**ABC** — is the union of the three closed segments **[AB], [BC]** and **[AC].**

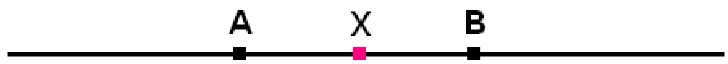
Here are some drawings for the first three items:

The drawing above depicts the line **AB.**

The drawing above depicts the closed line segment **[AB].**



The drawing above depicts the closed ray **[AB.**



In the drawing above, the statements **A∗X∗B** and **B∗X∗A** are both true, but the statements **A*B*X** and **X∗A∗B** are both false, and similarly the statements **X∗B∗A** and **B∗A∗X** are both false.

Sometimes it is also useful to have a concept of opposite ray:

9. Given two points **A** and **B**, the ***closed opposite ray to* [AB** is the set of all points on the line **[AB** but not on **(AB** and is denoted by **[AB**^{OP}**.** This can also be described as the set of all points **X** on **AB** such that **X∗A∗B** or **X∗A∗B** is true. Similarly, the ***open opposite ray*** **(AB**^{OP} is the set of all points **X** on **AB** such that **X∗A∗B** is true.

Here is a table relating the geometric concepts of segments and rays to the algebraic concept of an interval. In this table **f** denotes a real valued ruler function on **AB** such that **f(A) < f(B).**

Geometric object	Algebraic counterpart
line **AB**	$(-\infty, +\infty)$
closed segment **[AB]**	$[\,f(A), f(B)\,]$
open segment **(AB)**	$(\,f(A), f(B)\,)$
closed ray **[AB**	$[f(A), +\infty)$
open ray **(AB**	$(\,f(A), +\infty)$
closed opposite ray **[AB**^{OP}	$(-\infty, f(A)\,]$
open opposite ray **(AB**^{OP}	$(-\infty, f(A)\,)$

Next, we shall give the conventions involving angles (to be formally introduced later).

10. Given three noncollinear points **A, B** and **C,** the ***angle*** **∠ABC** is defined to be the union of the rays **[BC** and **[BA.** The ***measure*** of this angle, usually but not always expressed in degrees throughout the notes, is denoted by **|∠ABC|.**

IMPORTANT REMARKS. This definition **excludes** the extreme concepts of a ***zero – degree angle*** for which the two rays are equal and of a ***straight angle*** in which the two rays are opposite rays on the same line (and the points in question satisfy **A∗B∗C**).

It follows immediately from the definitions that **∠CBA = ∠ABC.** Note that ***the statement*** **∠ABC = ∠DEF** ***is much stronger than saying the two angles have the same measures*** (in symbols, **|∠ABC| = |∠DEF|**); it means that ***the two angles consist of exactly the same points.***