Solutions to exercises

1. L is the unique line containing x & y, x & z and y & z. Hence we may write $L = xy = xz = yz$ ∎

2. Since three collinear points $\Rightarrow$ exactly one between other two, if $X \in L = AB$ then exactly one of $X = A$, $X = B$, $A * X * B$, $A * B * X$, $X * A * B$ is true. By definition $X \in [AB \Leftrightarrow$ one of the first four is true. Since $(AB^{op} \Leftrightarrow X \in L$ but $X \notin [AB$, we are forced to conclude that $X \in (AB^{op} \Leftrightarrow X * A * B$ is true. ∎

3. (a) Choose a ruler function so that $f(B) > 0 = f(A)$. Then $X \in (AB \Leftrightarrow f(X) > 0$. By the description of $(AX$, it is all Y so that $f(Y) > 0$, and this is precisely the definition of $(AB$. ∎

3 (b) Choose f as before, let $g = -f$. Then $f(C) < 0 < f(A)$, so $(AB^{op} =$ all X s.t. $f(X) < 0 =$ all X s.t. $g(X) > 0$. Since $g(C) > 0$ this means

$X \in (AB^{op} \iff f(X) < 0 \iff g(X) > 0 \iff X \in (AC$

4. Case (a) Set is $[AB]$. Then $0 < f(X), f(Y) < f(B)$. Two subcases (i) $f(X) < f(Y)$. Then $X * Z * Y \Rightarrow f(X) < f(Z) < f(Y)$, so

$$f(A) \leq f(X) < f(Z) < f(Y) \leq f(B). \blacksquare$$

(ii) $f(Y) < f(X)$. Switch roles of X + Y in the preceding argument.

Case (b) Same idea but now $f(A) <$ both $f(X)$ + $f(Y)$ and $f(B) >$ both.

Case (c) Set is $[AB$, so $f(X), f(Y) \geq 0$. Two subcases as before: (i) $f(X) < f(Y) \Rightarrow$
$f(A) \leq f(X) < f(Z) < f(Y) \Rightarrow Z \in [AB$. (ii)
Switch X + Y to conclude the reverse case. $\blacksquare$

Note Similar conclusions true for $(AB$, $[AB^{op}$, $(AB^{op}$.