Math 133
Spring 2022
L04A

PROPERTIES OF TRIANGLE CONGRUENCE

Here are some details for statements from Lecture 04. We noted that $\triangle ABC \cong \triangle DEF$ does not necessarily imply statements like $\triangle ACB \cong \triangle DEF$; if both statements are true then we have $|AB| = |DE|$ and $|AC| = |DE|$, so that $|AB| = |AC$, and we know that there are triangles for which this equation is not valid. However, we noted that if the vertices of both triangles are rearranged in the same fashion, then one does get a valid new congruence statement' Here is a list of all the congruence statements that are equivalent to $\triangle ABC \cong \triangle DEF$:

$$\triangle ACB \cong \triangle DFE$$

$$\triangle BAC \cong \triangle EDF$$

$$\triangle BCA \cong \triangle EFD$$

$$\triangle CAB \cong \triangle FDE$$

$$\triangle CBA \cong \triangle FED$$

No other statements about rearrangements (for example $\triangle ABC \cong \triangle FED$) are valid for ALL triangles satisfying $\triangle ABC \cong \triangle DEF$.

And here are the details for verifying the equivalence properties of congruent triangles:

$\triangle ABC \cong \triangle ABC$: This is true because $|AB| = |AB|, |BC| = |BC|, |AC| = |AC|$, $|\angle BAC| = |\angle BAC|$, $|\angle ABC| = |\angle ABC|$, and $|\angle ACB| = |\angle ACB|$.$\blacksquare$

$\triangle ABC \cong \triangle DEF$ **implies** $\triangle DEF \cong \triangle ABC$: This is true because $|AB| = |DE|$, $|BC| = |EF|, |AC| = |DF|$, $|\angle BAC| = |\angle EDF|$, $|\angle ABC| = |\angle DEF|$, and $|\angle ACB| = |\angle DFE|$ imply $|DE| = |AB|, |EF| = |BC|, |DF| = |AC|$, $|\angle EDF| = |\angle BAC|$, $|\angle DEF| = |\angle ABC|$, and $|\angle DFE| = |\angle ACB|$.$\blacksquare$

$\triangle ABC \cong \triangle DEF$ **and** $\triangle DEF \cong \triangle GHK$ **imply** $\triangle ABC \cong \triangle GHK$: This is true because we have $|AB| = |DE| = |GH|, |BC| = |EF| = |HK|, |AC| = |DF| = |GK|$, $|\angle BAC| = |\angle EDF| = |\angle HGK|$, $|\angle ABC| = |\angle DEF| = |\angle GHK|$, and $|\angle ACB| = |\angle DFE| = |\angle GKH|$.$\blacksquare$