Math 133
Fall 2021

EXERCISES FOR WEEK 03

Assume that $(\mathbf{S}; \mathcal{P}; \mathcal{L}; d; \alpha)$ or $(\mathbf{P}; \mathcal{L}; d; \alpha)$ is a system which satisfies the Incidence, Ruler, Plane or Space Separation (as appropriate), Angle Measurement and Triangle Congruence Axioms; in other words, the system is a **neutral geometry**.

1. Suppose we are given triangle $\triangle ABC$, and assume that the two angle measures $|\angle BCA|$ and $|\angle CAB|$ are less than $90°$. Let $D \in AC$ be such that BD is perpendicular to AC. Prove that D lies on the open segment (AC). [*Hint:* The Exterior Angle Theorem is needed. Why can we eliminate all possibilities except $A * D * C$?]

2. Suppose we are given triangle $\triangle ABC$. Prove that at least one of the following three statements is true:

(1) The perpendicular from A to BC meets the latter in (BC).

(2) The perpendicular from B to CA meets the latter in (CA).

(3) The perpendicular from C to AB meets the latter in (AB).

[*Hint:* How many angles in a triangle are either right or obtuse?]

Sketch an example where all three statements are true, and sketch an example where only one statement is true.

3. (a) Suppose that we are given $\angle ABC$ such that $[BD$ bisects $\angle BAC$, and let $E \in (BA$ and $F \in (BC$ be such that $|\angle DEB| = |\angle DFB|$. Prove that $|DE| = |DF|$. [*Note:* At this point we do not know whether or not the angle sum of a triangle is $180°$.]

(b) Suppose we are given isosceles triangle $\triangle ABC$ with $|AC| = |BC|$, and we have points $D \in (AC)$, $E \in (BC)$ such that $|AD| < |BE|$. Prove that $|\angle ADE| < |\angle BED|$.

4. Given $\triangle ABC$, let X, Y and Z be points on the open segments (AB), (BC) and (AC) respectively; by a previous exercise, we know these points are not collinear and hence form the vertices of a triangle. Prove that the sum of the lengths of the sides of $\triangle ABC$ is greater than the sum of the lengths of the sides of $\triangle XYZ$.

5. Let X be a point in the plane P. Prove that there is a pair of perpendicular lines L and M in P which meet at X and that there is no line N in P through X which is perpendicular to both L and M.

6. (*Half of Euclid's Fifth Postulate*) Let AB be a line, and let C and D be points such that A, B, C and D are coplanar and both C and D lie on the same side of AB. Prove the following:

(a) If the open rays $(AC$ and $(BD$ have a point in common, then $|\angle CAB| + |\angle DBA| < 180°$. [*Hint:* Why is there a triangle $\triangle ABE$ with $E \in (AC \cap (BD$?]

(b) If the lines AC and BD meet at a point on the side of AB which is opposite the side containing C and D, then $|\angle CAB| + |\angle DBA| < 180°$.