Math 133
Fall 2021

EXERCISES FOR WEEK 04

For the these exercises assume that $(\mathbf{S}; \mathcal{P}; \mathcal{L}; d; \alpha)$ or $(\mathbf{P}; \mathcal{L}; d; \alpha)$ is a system which satisfies the axioms for Euclidean geometry.

1. Suppose we are given a parallelogram $\diamondsuit ABCD$, and let $E \in (AB)$ and $F \in (CD)$ be such that $[DE$ bisects $\angle CDA$ and $[BF$ bisects $\angle ABC$. Prove that the lines DE and BF are parallel. [Why are the two lines distinct?]

2. Assume that we are given two angles $\angle DAB$ and $\angle DCB$ in a Euclidean plane such that $|\angle CAB| = |\angle DCB|$, and also assume that we have $X \in (AB) \cap (CD)$. Prove that $|AX| \cdot |XB| = |CX| \cdot |XD|$.

3. (a) Suppose that $\triangle ABC$ satisfies $\triangle ABC \sim \triangle BCA$ (note the orderings of the vertices!). Prove that $\triangle ABC$ is equilateral. [*Hints:* Why do we also have $\triangle BCA \sim \triangle CAB$ and $\triangle CAB \sim \triangle ABC$? with the same ratio of similitude? If $\triangle ABC \sim \triangle ABC$, why is 1 the only possible ratio of similitude?]

(b) Suppose that $\triangle ABC$ satisfies $\triangle ABC \sim \triangle ACB$ (note the orderings of the vertices!). Prove that $\triangle ABC$ is isosceles, and give an example to show that this triangle need not be equilateral.

4. Given $\triangle ABC$, let D and E denote the midpoints of $[AB]$ and $[AC]$ respectively. Prove that DE is parallel to AB and $|DE| = \frac{1}{2}|BC|$. [*Hint:* Show that the line through D which is parallel to BC contains E.]

5. Suppose that we are given positive real numbers $a_1, , a_n$ and $b_1, , b_n$ such that

$$\frac{a_1}{b_1} = \frac{a_i}{b_i}$$

for all i. Prove that

$$\frac{a_1}{b_1} = \frac{a_1 + ... + a_n}{b_1 + ... + b_n} .$$

6. Let $\triangle ABC$ be given, and let Z be a point on $(AB$. Let X and Y be points on the same side of AB as C such that AX, CZ and BY are all parallel to each other, and also assume that $B * C * X$ and $A * C * Y$. Prove that

$$\frac{1}{|CZ|} = \frac{1}{|AX|} + \frac{1}{|BY|} .$$