Math 133
Fall 2021

EXERCISES FOR WEEK 05

For the these exercises assume that $(\mathbf{S}; \mathcal{P}; \mathcal{L}; d; \alpha)$ or $(\mathbf{P}; \mathcal{L}; d; \alpha)$ is a system which satisfies the axioms for Euclidean geometry. Background material on circles is given in Chapter 16 of Moise.

1. Prove Theorems 4, 6 and 7 on page 227 of Moise.

2. Let Γ be a circle with radius r, and let A and B be points on the circle. Prove that a point $X \in AB$ lies in the interior of the circle if $A * X * B$, and X lies in the exterior of the circle if either $X * A * B$ or $A * B * X$. [*Hints:* If the center Q of Γ does not lie on AB, consider $\triangle QAB$, and look at the implications of (i) the Exterior Angle Theorem, (ii) the theorem stating that the longer of two edges of a triangle is opposite the larger angle.]

3. Suppose that we are given four points A, B, C, D on a circle Γ, whose center is Q, such AD and BC meet at Q. Prove that the lines AB and CD are parallel. [*Hint:* Note that Q is the common midpoint of $[AC]$ and $[BD]$ and look for pairs of alternate interior angles.]

4. Use coordinates to prove the following two statements:

(a) Three distinct points on a circle cannot be collinear.

(b) Two distinct circles have at most two points in common.

5. Let Γ be a circle in the plane, let A be a point in the interior of Γ and let X be a point different from A. Prove that the ray $[AX$ meets Γ in exactly one point. [*Hint:* By the line–circle theorem, the line AX meets Γ in two points B and C. Why do these points lie on opposite rays?]