Math 133
Spring 2022
L11B

Clarification for page 2 of `lecture11.pdf`

In the argument showing tha the lines AD, BE and CF have a point in common, the last two lines might note be sufficiently transparent, so here are the details. In the first part of the proof we showed that the point G where AD meets CF can be written in vector form as $\frac{1}{3}(\beta+\gamma)$ because we are assuming $\alpha = 0$. To complete the proof we need to show that the lines BE and CF also meet at this point. To find the point H where these two lines meet, we need to find r and s such that the following vector equation is satisfied:

$$H \quad = \quad \beta + r \cdot \left(\frac{1}{2}(\alpha+\gamma) - \beta\right) \quad = \quad \gamma + s \cdot \left(\frac{1}{2}(\alpha+\beta) - \gamma\right)$$

Simplifying this (and recalling that $\alpha = 0$) we obtain the following:

$$= \quad (1-r)\beta + \frac{r}{2}\gamma \quad = \quad (1-s)\gamma + \frac{s}{2}\beta$$

Equating the coefficients of β and γ in these expressions, we obtain $\frac{1}{2}r \quad = \quad 1-s$ and $\frac{1}{2}s \quad = \quad 1-r$. The unique solution for this system is $r = s = \frac{1}{3}$ and therefore $H = \frac{1}{3}(\beta+\gamma) = G$, so that the three lines all go through the point $\frac{1}{3}(\beta+\gamma)$.■