Math 133
Fall 2021

SOLUTIONS FOR WEEK 07 EXERCISES

For these exercises assume that the coordinate plane satisfies the axioms for Euclidean plane geometry, with distances and lines as described in Chapter 17 of Moise.

1. Let P be the set $\{A, B, C, D\}$ and let $\mathcal{L}$ consist of the subsets $\{A, B\}$, $\{A, C\}$, $\{A, D\}$ and $\{B, C, D\}$. One can then check that each line contains at least two point, and for each of the six unordered pairs $\{U, V\} \subset P$ there is exactly one subset in $\mathcal{L}$ which contains the given pair.■

2. Take any of the systems listed in the hint with the designated binary operations. Since the new addition agrees with the old one, the first four properties involving addition are automatically true (we are assuming all the properties hold for the number systems listed in the exercise). It remains to verify the commutativity and associativity of the circle action and to show that this operation does not satisfy the distributive law.

The commutative law follows from the chain of equations

$$a \circ b \ = \ a + b + ab \ = \ b + a + ba \ = \ b \circ a$$

and the associative law is a consequence of the following chain of equations:

$$a \circ (b \circ c) \ = \ a + (b + c + bc) + a(b + c + bc) \ = \ a + b + c + ab + bc + ac + abc \ =$$

$$(a + b) + c + ac + bc + ab + abc \ = \ (a + b + ab) + c + (ac + bc + abc) \ = \ (a \circ b) \circ c$$

To show that the distributive law is false, we compute $a \circ (b + c)$ and $(a \circ b) + (a \circ c)$ separately.

$$a \circ (b + c) \ = \ a + b + c + ab + ac$$

$$(a \circ b) + (a \circ c) \ = \ a + b + ab + a + c + ac \ = \ 2a + b + c + ab + ac$$

The right hand sides of these expressions are unequal if $a \neq 0$, and hence the distrbutive law does not hold for the $+$ amd $\circ$ operations. This proves that a system which satisfies all but the last axiom will not necessarily satisfyy the last one (the distribtive law).■