## UPDATED GENERAL INFORMATION — DECEMBER 4, 2013

**Information about the final examination.** As suggested previously, part of the examination will cover Sections III.4–III.6 and V.2–V.4 and part will cover earlier material from the course. As usual in mathematics courses, background from earlier parts of the course is likely to appear in the problems involving new material; furthermore, the old material may include some items from the "new" sections that could have easily been covered earlier (but the contents of the file `neutral-proofs1.pdf` fits into the "new" rather than the "old" category).

There will be seven problems with point values ranging from 15 to 25 points and a total of 150 points possible. New material will constitute $56\frac{2}{3}$ per cent of the point total, and specifically material from Unit V will constitute 30 per cent of the point total. Previous statements about the examination requiring only 90 minutes to complete may be too optimistic, and 2 hours might be a more realistic estimate. In any case, students have the entire 3 hour period to work on the examination.

Here is a rough breakdown into the types of questions; some problems on the exam may have parts of different types.

(1) **Statements of definitions, assumptions or results.** (35 points)

(2) **Proving statements at the level of fairly short proofs in the notes or relatively uncomplicated homework exercises.** In some cases synthetic methods are necessary or more efficient, and in other cases vector and algebraic methods are probably more efficient. Reasons may be given less formally if the problem asks for an explanation rather than a proof, but reasons are still needed for full credit. (90 points)

(3) **Deriving simple formulas which illustrate the basic definitions, assumptions or results.** (25 points)

At the beginning of almost every question there will be a statement whether the underlying geometry is assumed to be **Euclidean**, **hyperbolic** or **neutral**. If there is no such statement, a proof for any one or more of these cases will be acceptable.

Here are more specific suggestions, including some problems which were considered but either not included or may appear in a simplified form.

(1) Assume that we are working in a given **Euclidean** plane. — Suppose that $\Delta ABC$ is isosceles with $d(A,B) = d(A,C)$, and let $D$ be the midpoint of $[BC]$. Explain why each of the centroid, circumcenter, orthocenter and incenter lie on the line $AD$. Describe examples for which all four of these points lie on $[AD]$, and also describe examples for which at least one of these points does not lie on the open segment $(AD)$.

(2) Assume that we are working in a given **Euclidean** plane. — Suppose that we are given $\Delta ABC$, and let $D \in (AB)$ be such that $\Delta ADC \sim \Delta CDB$. Prove that $\Delta ACB$ is a right triangle, and furthermore we have $\Delta ADC \sim \Delta ACB$ and $\Delta CDB \sim \Delta ACB$.

(3) Assume that we are working in a given **Euclidean** plane. — Suppose that we are given $\Delta ABC$ and $\Delta DEF$, and also assume that there are points $G \in (BC)$, $H \in (EF)$ such that $\Delta ABG \sim \Delta DEH$ and $\Delta AGC \sim \Delta DHF$. Prove that $\Delta ABC \sim \Delta DEF$.

(4) Assume that we are working in a given **Euclidean** plane. — Suppose that we are given lines $L$ and $M$ with distinct points $A, B, C, D, E, F$ such that the first three points are on $L$ and satisfy $A * B * C$, while the second three points are on $M$ and satisfy $D * E * F$. Furthermore, assume that the three lines $AD$, $BE$ and $CF$ are all parallel to each other. Prove that

$$\frac{d(A,B)}{d(B,C)} \quad = \quad \frac{d(D,E)}{d(E,F)} \ .$$

[*Hint:* Use vectors and write $C = A + t(B - A)$ and $F = D + u(E - D)$ for suitable scalars. What do the betweenness relations say about $t$ and $u$ separately? How do the parallelism conditions imply an equation involving $t$ and $u$?]

Also consider the following converse problem: If two of the three lines are parallel and the proportionality equation is valid, prove that the third line is parallel to the other two.

(5) Assume that we are working in a given **neutral** plane. — Suppose that $\Delta ABC$ is isosceles with $d(A,B) = d(A,C)$, let $E$ and $F$ be the midpoints of $[AB]$ and $[AC]$ respectively, and suppose that $D$ is the midpoint of $[BC]$. Prove that the lines $AD$ and $EF$ are perpendicular. [*Note:* At some point in the proof it will probably be necessary to show that $AD$ and $EF$ have a point in common.]

(6) Assume that we are working in a given **neutral** plane. — Suppose that $\Delta ABC$ is given and that $B * C * D$ is true. Prove that $|\angle BCD| \geq |\angle BAC| + |\angle ABC|$. Can we state a stronger conclusion if the plane is hyperbolic? Give reasons for your answer.

(7) Assume that we are working in a given **hyperbolic** plane. — Using the exercises for Section V.3 and results from Section V.4, explain why a Saccheri quadrilateral is never a Lambert quadrilateral and vice versa. [*Hint:* In neutral geometry, explain how the exercises imply that a convex quadrilateral which is both a Saccheri quadrilateral and a Lambert quadrilateral must be a rectangle.]

(8) Suppose we are given a circle $\Gamma$ in a **neutral** plane, and suppose that $A, B, C \in \Gamma$ are such that neither the line $AB$ nor the line $BC$ contains the center $Q$ of the circle. Prove that $Q$ lies on the perpendicular bisectors of $[AB]$ and $[BC]$.

(9) Assume that we are working in some **Euclidean** plane. — Suppose that we are given real numbers $0 < a < b$. Explain why there is a triangle in the plane whose sides have length $a$, $2a$ and $b$ if and only if $b < 3a$.