FURTHER COMMENTS ON EXAMINATION 2

Problem 1. One can also prove this result using the **AAS** congruence criterion as follows: By the first triangle congruence given in the problem we have $d(A, B) = d(D, E)$ and $|\angle ABC = \angle ABG| = |\angle DEH = \angle DEF|$, and by the second triangle congruence given in the problem we have $d(A, C) = d(D, F)$ and $|\angle ACB = \angle ACG| = |\angle DFH = \angle DFE|$. In fact, we have **SAAS** for $\triangle ABC$ and $\triangle DEF$.

Another possible approach to the problem is to show that $|\angle BAC| = |\angle EDF|$ and apply the **SAS** congruence criterion. ***However***, in order to carry out this approach, it is necessary to begin by checking **explicitly** that G lies in the interior of $\angle BAC$ and H lies in the interior of $\angle EDF$. Both of these can be derived from the assumptions that G and H lie on (BC) and (EF) as follows: The given conditions imply that **(1)** G and C lie on the same side of AB, **(2)** G and B lie on the same side of AC, **(3)** H and E lie on the same side of DF, **(4)** H and F lie on the same side of DE. If we combine **(3)** and **(4)** we find that G lies in the interior of $\angle BAC$, and if we combine **(3)** and **(4)** we find that H lies in the interior of $\angle EDF$. With these conclusions at our disposal, we can quickly prove the desired equality of angle measures with two applications of the Additivity Postulate for angle measurements as follows:

$$|\angle BAC| = |\angle BAG| + |\angle GAC| = |\angle EDH| + |\angle HDF| = |\angle EDF|$$

Finally, although one can roughly summarize the exercise as saying that $\triangle ABG \cong \triangle DEH$ and $\triangle AGC \cong \triangle DHF$ imply that

$$\triangle ABC = \triangle ABG + \triangle AGC \cong \triangle DEH + \triangle DHF = \triangle DEF$$

a statement of this type cannot be used as a valid reason in a proof of the conclusion. The simplest reason is that the sum of two triangles with a common edge was not formally defined, and an even more important reason is that a statement of the form, "*if congruent triangles be added to congruent triangles, then their sums are congruent*," is essentially what we want to prove, and thus any argument which uses such a principle is almost certain to be circular.

Problem 2. One key observation in this exercise is that $|\angle BAC| < 60°$ implies that the remaining two base angles of the triangle satisfy $|\angle ABC| = |\angle ACB| > 60°$. Strictly speaking, a formal algebraic proof is required to justify this, but there were no penalties for a lack of a detailed algebraic argument in examination answers because there was another more crucial point at which a more formal deductive step was needed (to conclude that the longer side is opposite the larger angle) and one goal of the problem was to test the ability to figure out what the correct answer should be. In other situations, it is likely that points would be deducted if such algebraic details were missing.

Problem 5. Ordinarily formulas with vectors in the denominator are not valid because one rarely has criteria for dividing one vector by another; in particular, there is no way of doing such things legitimately with $3-$dimensional vectors. However, since the problem involved $2-$dimensional vectors and one can justify writing such fractional expressions using the arithmetic of complex numbers, an answer to this problem involving vectors in the denominator was acceptable (but normally ***such notation should not be employed*** because it can lead to meaningless conclusions).