NAME: ______________________________

Mathematics 133, Fall 2007, Examination 1

Answer Key

1. [*25 points*] Find two points on the line L given by the intersection of the following two planes:

$$x - y = 2 , \qquad y - z = 5$$

SOLUTION

There are several valid approaches. For example, one can do so using row operations on the augmented matrix:

$$\begin{pmatrix} 1 & -1 & 0 & : & 2 \\ 0 & 1 & -1 & : & 5 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -1 & : & 7 \\ 0 & 1 & -1 & : & 5 \end{pmatrix}$$

We can then solve for x and y in terms of z to get $x = 7 + z$, $y = 5 + z$. Any two points expressible in this form will yield a valid answer.

2. [*20 points*] (*a*) State the algebraic condition on vectors **x**, **y**, **z** in $\mathbf{R}^n$ which is equivalent to the additivity equation for distances:

$$d(\mathbf{x},\,\mathbf{z}) \ = \ d(\mathbf{x},\,\mathbf{y}) + d(\mathbf{y},\,\mathbf{z})$$

(*b*) Suppose that **a** and **b** are two points in the plane or space, and let suppose that **x** is a third point which lies on the line **ab**. Let C be the set of all points that are on the ray [**ab** but not on the closed segment [**ab**]. State the betweenness condition on **x, a, b** which is equivalent to saying that **x** lies on C.

SOLUTION

(*a*) The **algebraic** condition is that $\mathbf{y} = t\mathbf{x} + (1-t)\mathbf{z}$ where $0 \leq t \leq 1$. The geometric condition is that **y** should be EITHER between the other two points OR equal to one of them.

(*b*) Since **x** lies on the ray [**a**, **b** if and only if **x** is one of **a** or **b**, **x** is between **a** and **b**. or **b** is between **a** and **x**, and **x** lies on the segment [**a**, **b** if and only if **x** is one of **a** or **b**, or **x** is between **a** and **b**, the condition for **x** to lie on X is simply the condition that **b** must be between **a** and **x**. In other words, **x** must lie on the open ray $(\mathbf{b}\,\mathbf{a}^{\mathbf{OP}}$.

3. [*30 points*] Find the barycentric coordinates of $(2,2)$ with respect to $(1,0)$, $(5,0)$ and $(1,4)$.

SOLUTION

Call the first point D and denote the remaining points by A, B and C respectively. We need to find scalars v and w such that $D - A = v(B - A) + w(C - A)$. Substituting in the numerical values, we obtain the following equation(s):

$$(1,2) \quad = \quad v\,(4,0) \; + \; w\,(0,4)$$

If we solve these for v and w we obtain $v = \frac{1}{4}$ and $w = \frac{1}{2}$. It follows that if

$$D \quad = \quad u\,A \; + \; v\,B \; + \; w\,C$$

with $u + v + w = 1$, then we must have $D \;=\; \frac{1}{4}\,A \;+\; \frac{1}{4}\,B \;+\; \frac{1}{2}\,C$.

4. [*25 points*] Given the three points $(2, 5)$, $(6, 5)$ and $(6, 2)$, find two of them which lie on the same side of the line defined by the equation $3y - 2x = 1$.

SOLUTION

Let $f(x, y) = 3y - 2x - 1$. Then we have $f(2, 5) > 0$, $f(6, 5) > 0$ and $f(6, 2) < 0$. Therefore $(6, 5)$ and $(2, 5)$ lie on one side of the line, while $(6, 2)$ lies on the other,