SOLUTIONS TO ADDITIONAL EXERCISES ON CONCURRENCE

Here are the solutions to the exercises in `concurrence.pdf`.

6. If V is the circumcenter of $\Delta\, ABC$, then one can find V by solving the system of equations

$$|V - A|^2 \ = \ |V - B|^2 \ = \ |V - C|^2 \ .$$

If we let $V = (x, y)$ and substitute for $A,\ B,\ C$, then we obtain the equations $x + y = 6$ and $6x - 2y = 14$. The solution to this system of equations is $x = \frac{13}{4}$ and $y = \frac{11}{4}$.■

7. The equation of the line M which is perpendicular to L (with equation $y = 2x+2$ and passes through $(1, 0)$ must have the form $y = -\frac{1}{2}x + C$ for some constant C. Since the line passes through $(1, 0)$ it follows that the constant must be $\frac{1}{2}$. Now the orthocenter is is a point which lies on all three altitudes, so it is enough to find a point where two of the altitudes meet. Since the altitude from $(0, 2)$ is just the y-axis, the point in question turns out to be $(0, \frac{1}{2})$.■

Alternate approach. Another way to find the orthocenter of a triangle $\Delta\, ABC$ is to find the circumcenter of the associated triangle $\Delta\, DEF$, where $D = C + B - A$, $E = A + C - B$, and $F = A + B - C$; recall that DE is the unique line through C which is parallel to AB, while EF is the unique line through A which is parallel to BC and DF is the unique line through B which is parallel to AC. — For the specific example in this exercise, we may take $A = (0, 2)$, $B = (-1, 0)$ and $C = (1, 0)$, so that $D = (0, -2)$, $E = (2, 2)$ and $F = (-2, 2)$. The equations for the circumcenter of $\Delta\, DEF$ reduce to $8x = 0$ and $4x + 8y = 4$, so that $x = 0$ and $y = \frac{1}{2}$. This is the same conclusion that was obtained in the preceding paragraph.■