

## SUMMARIZING THE IMPACT OF NON – EUCLIDEAN GEOMETRY

**NOTE.** *The following material should be viewed as an additional section (Section V.8) to be added at the end of the file geometrynotes5b.pdf.*

The last few sections of Unit V discuss the impact of non – Euclidean geometry on mathematics and related subjects from several different viewpoints. The following list summarizes the discussion:

1. In contrast to the laws of physics and chemistry, the laws for whole number arithmetic seem to be logically inevitable. However, the development of non – Euclidean geometry during the late 18<sup>th</sup> and most of the 19<sup>th</sup> centuries showed very conclusively that the classical setting for (Euclidean) geometry was not equally logically inevitable.
2. In many respects, the classical non – Euclidean geometry, which is often called *hyperbolic geometry*, behaves like a spherical geometry in which the radius of the sphere is an imaginary number.
3. Non – Euclidean geometry was one of several important factors leading to the creation of a more rigorous logical foundation for mathematics during the late 19<sup>th</sup> and early 20<sup>th</sup> centuries. Mathematical models for both Euclidean and non – Euclidean geometry can be constructed within these frameworks for mathematics.
4. The development of non – Euclidean geometry eventually led to many alternative mathematical theories for describing the physical universe, including the currently standard models of Relativity Theory. There are some relationships between the latter and non – Euclidean geometry, but neither can really be described part of the other.
5. For many (maybe most) practical purposes, Euclidean geometry is an extremely good first order approximation for studying small regions in any reasonable theory of space.
6. Non – Euclidean geometry has turned out to be more than just a logical curiosity, and many of its basic features continue to play important roles in several branches of mathematics and its applications. Two simply stated examples involve the proof of Fermat’s Last Theorem and several of Maurits C. Escher’s artistic creations.