

UPDATED GENERAL INFORMATION — NOVEMBER 5, 2007

NEW DUE DATE FOR HOMEWORK ASSIGNMENT. The assignment will now be due in discussion the section meeting on **Thursday, November 8.**

QUIZ COVERAGE. The quiz on November 8 will cover material from Sections II.3 through III.2.

STEPS IN A PROOF OF THEOREM III.2.13. This is a slightly different approach than the one in the notes, and we concentrate on the steps involving order and separation.

For this argument, we are given $\triangle ABC$, and L is the unique line through A which is parallel to BC . Points D and E on L are chosen so that C and D lie on opposite sides of AB , and E is chosen so that $D * A * E$. — The objective is to prove (i) B and E lie on opposite sides of AB , (ii) the point B lies in the interior of $\angle DAC$.

If these two conditions are known, they are enough to justify the usual proof that the sum of the measures of the vertex angles in $\triangle ABC$ is 180 degrees. We shall go through the argument step by step.

1. There is a point $X \in AB \cap (CD)$ (by the Plane Separation Property and the assumption that C and D lie on opposite sides of AB).
2. The points B and C lie on the same side of L (by construction $BC \parallel L$, and if they were on different sides then (BC) and L would have a point in common).
3. X and C lie on the same side of L ($C \notin L$ implies that $CD \neq L$, so $C * X * D$ and $D \in L$ imply C and X lie on the same side).
4. X and B lie on the same side of L (combine the preceding two steps).
5. $[AX = [AB$ (since $[AY$ is the union of $\{A\}$ with all points on the same side of L as Y in each case).
6. X and B lie on the same side of AC (since $[AX = [AB$ is the union of $\{A\}$ with all points on one side of AC).
7. X and D lie on the same side of AC (since $C * X * D$ and $CD \neq AC$, the latter because $C \notin L = AD$).
8. B and D lie on the same side of AC (combine the preceding two steps).
9. **THEREFORE** B and E lie on opposite sides of AC ($D * A * E$ implies that D and E lie on opposite sides, while the preceding step shows B and D lie on the same side).
10. **FURTHERMORE** B lies in the interior of $\angle DAC$ (by Step 2 we know that B and C lie on the same side of $AD = L$, while by Step 8 we know that B and D lie on the same side of AC). ■