

Readings for Unit II (Vector algebra and Euclidean geometry)

II.1 : Approaches to Euclidean geometry

Supplementary background readings.

Ryan : pp. 5 – 15

Comments.

The passage begins with the subheading, “Three approaches to the study of geometry,” on page 5, and it continues up to (but not including) the subheading, “Orthonormal pairs,” on page 15. In the first few pages there is a discussion of different approaches to geometry that is somewhat complementary to the corresponding discussion in the notes, and the remaining material includes a discussion of the analytic approach in a manner similar to the course notes.

II.2 : Synthetic axioms of order and separation

Supplementary background readings.

Ryan : pp. 11 – 15, 50 – 52, 58 – 62, 68

Comments.

The first passage includes the algebraic formulation of the geometrical notion of betweenness for triples of collinear points, while the second passage is the subheading “Rays and angles,” and its approach is somewhat more analytic than the approach in the course notes. In the third passage, barycentric coordinates are applied to study the Plane Separation Property and related results which play an important role in the course notes, and once again the treatment is somewhat more analytic in places. This passage ends just before the subheading, “Symmetries of a triangle.” Finally, several exercises related to this section of the course notes are given on page 68.

II.3 : Measurement axioms

Supplementary background readings.

Ryan : pp. 11 – 15, 50 – 52, 58 – 62, 68

Comments.

The first passage contains basic definitions (as before), while the second passage, which is the subheading “Rays and angles,” provides a different perspective on the theory of angle measurement. In the third passage, which starts with the subheading “Barycentric coordinates” and goes through the end of the subheading “Triangles,”

there is a discussion of sums of angles which complements the course notes. Page 68 contains exercises related to this section of the course notes.

II.4 : Congruence, superposition and isometries

Supplementary background readings.

Ryan : pp. 11 – 15, 19 – 21, 49 – 52, 55 – 62, 64 – 66, 68

Comments.

The comments for the previous section also apply here to the first passage, the subheadings “Barycentric coordinates,” “Triangles” and “Congruence theorems for triangles” on pages 58 – 60, 61 – 62 and 65 – 66, and likewise to the exercises on the final page. In the other pages, the book studies the concept of *isometry* (which is introduced in the notes) from a somewhat different viewpoint, with particular emphasis on its ties to the intuitive notion of symmetry for certain familiar geometrical figures. A symmetry is an isometry that is a $\mathbf{1} - \mathbf{1}$ correspondence from a give figure to itself. In the subheading “Reflections,” the appropriate symmetries are reflections of a plane about a line; physically speaking, these correspond to taking the mirror image of a point with respect to a specified line in the plane. The subheading “Symmetry groups” discusses an important symmetry property of isosceles triangles which is also covered in the notes but from a slightly different viewpoint. Another class of isometries — namely, the mathematical analog of a physical rotation — plays a very important role in Ryan’s formulation of some key concepts involving angular measurement on pages 50 – 52 and 64 – 65. The subheadings, “Symmetries of a segment” and “Symmetries of an angle,” which appear on pages 55 – 58, give a formal description of the intuitively apparent reflection symmetries of a closed line segment with respect to its midpoint and of an angle with respect to the line of it bisector. In the subheading “Affine symmetries” which appears on pages 49 – 50, there is a corresponding analysis involving affine transformations instead of isometries in certain cases. Finally, the addition and supplementary angle properties for angle measurements, which appear in the subheading “Addition of angles” on pages 60 – 61, are derived using rotations.

II.5 : Euclidean parallelism

Supplementary background readings.

Ryan : pp. 11 – 15, 17 – 18, 49 – 50, 58 – 59, 68

Comments.

The comments for Section **II.3** also apply here.