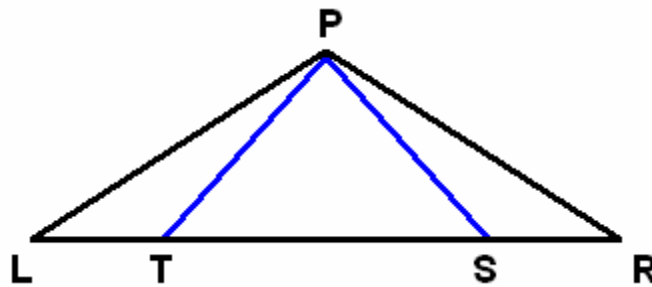


# FIGURES FOR SOLUTIONS TO SELECTED EXERCISES

## III : Basic Euclidean Concepts and theorems

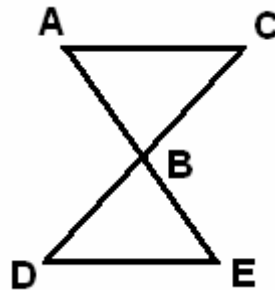
### III.2 : Basic theorems on triangles

III.2.3.



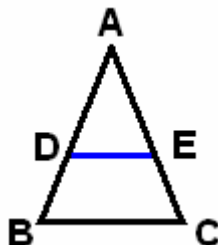
We are given  $R*S*T$ ,  $d(R, S) = d(L, T)$ , and  $d(P, S) = d(P, T)$ . The objective is to prove that we have overlapping congruent triangles  $\triangle RTP \cong \triangle LSP$  and that we also have  $|\angle PSR| = |\angle PTL|$ .

III.2.4.



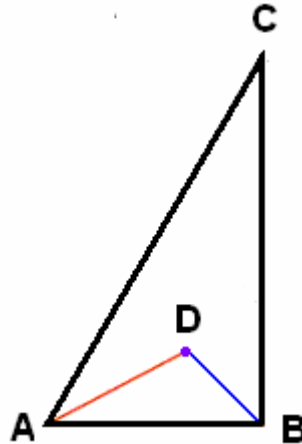
The point **B** which is the midpoint of both **[AE]** and **[CD]**, and the objective is to prove that **AC**  $\parallel$  **DE**. One method for doing this is to find a pair of alternate interior angles.

III.2.6.



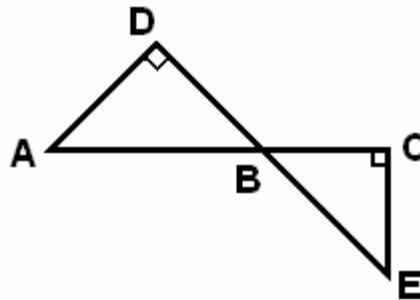
We are given that  $d(A, B) = d(A, C)$ , and that  $D$  and  $E$  are points of  $(AB)$  and  $(AC)$  such that  $d(A, D) = d(A, E)$ . To prove that  $BC \parallel DE$ , it suffices to find a pair of corresponding angles.

### III.2.7.



More generally, if  $D$  is a point in the interior of  $\triangle ABC$  such that  $[AD$  bisects  $\angle CAB$  and  $[BD$  bisects  $\angle CBA$ , then there is a formula relating  $|\angle ADB|$  and  $|\angle ACB|$ .

### III.2.8.



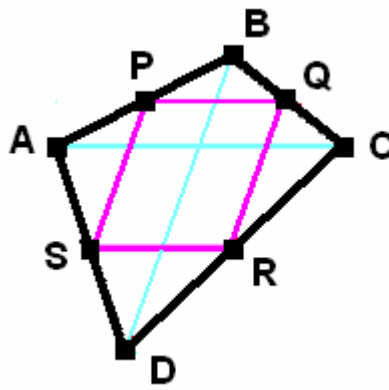
The perpendicular pairs of lines are marked in the drawing, and the objective is to show that  $|\angle DAB| = |\angle BEC|$ . In high school geometry books, such a result is often stated in the form ***if two angles have their corresponding sides perpendicular, left to left and right to right, then the angles have equal measurements.***

# FIGURES FOR SOLUTIONS TO SELECTED EXERCISES

## III : Basic Euclidean Concepts and theorems

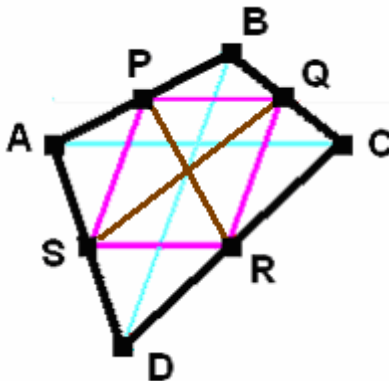
### III.3 : Convex polygons

#### III.3.1.



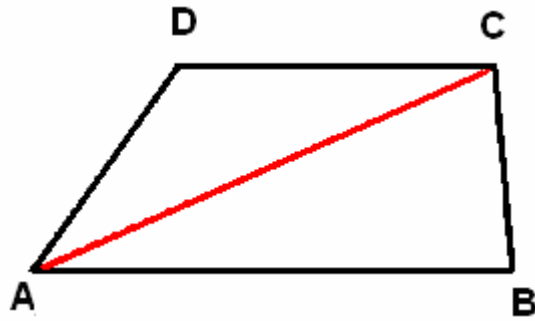
In this figure, each of the diagonals of the convex quadrilateral determines a pair of triangles which share a common edge. The midpoints of the sides of the original convex quadrilateral are also midpoints of the sides of these triangles.

#### III.3.2.



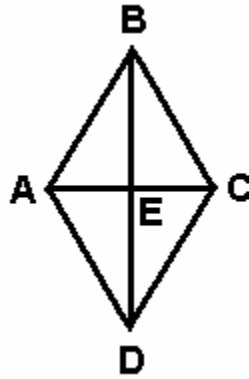
By a previous exercise we know that the midpoints **P, Q, R, S** (in that order) determine a parallelogram.

II.3.4.



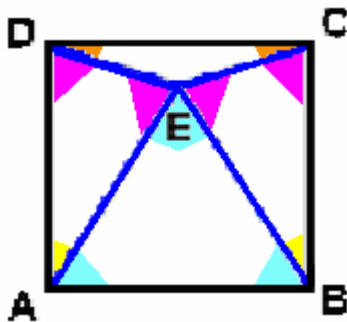
We are given that  $d(A, D) = d(C, D)$  in the trapezoid illustrated above, and the objective is to show that  $AC$  bisects  $\angle DAB$ . Note that  $\triangle DAC$  is isosceles.

III.3.6.



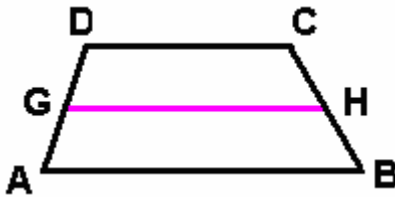
The points  $A, B, C, D$  form the vertices of a parallelogram, and the objective is to show that this parallelogram is a rhombus (all sides have equal length) if and only if  $AC$  is perpendicular to  $BD$ .

II.3.7.

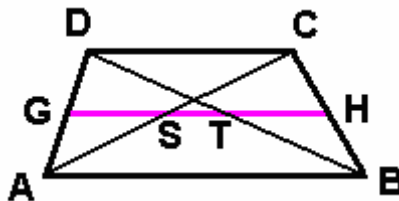


In the drawing, two angles have equal measurement if they are marked with the same color.

### III.3.14.

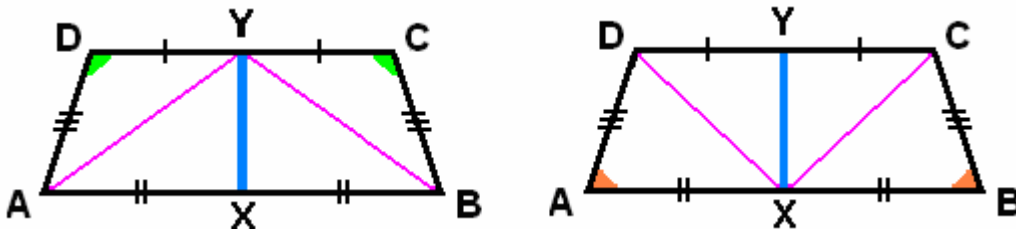


The points **A, B, C, D** form the vertices of a trapezoid with  $AB \parallel CD$ , and the midpoints of  $[AD]$  and  $[BC]$  are **G** and **H** respectively. In the first part of this exercise, the objective is to show that **GH** is parallel to **AB** and **CD** and to find its length in terms of the lengths of  $[AB]$  and  $[CD]$ .



In the final part of the exercise the objective is to show that the midpoints **S** and **T** of the diagonals  $[AC]$  and  $[BD]$  both lie on the line **GH**.

### III.3.15.



We are assuming that the points **A, B, C, D** form the vertices of an *isosceles* trapezoid with  $AB \parallel CD$ , and the points **X** and **Y** are the midpoints of  $[AB]$  and  $[CD]$  respectively. The idea is to use the basic properties of an isosceles trapezoid (various parts have equal measurements) to show that  $\triangle ADY \cong \triangle BCY$  and  $\triangle XAD \cong \triangle XBC$ , so that **Y** will be equidistant from **A** and **B**, and **X** will be equidistant from **C** and **D**.