Passage from the synthetic to the analytic approach

(excluding areas and volumes)

This requires a proof that an abstract system of data satisfying the axioms in the first two groups must look like the analytic construction; in other words, there is a $1-1$ correspondence from the abstract plane **P** or space **S** to coordinate space of 2 or 3 dimensions such that synthetic lines and planes correspond to analytically defined lines and planes, the synthetic distance is given by the analytic formula, and likewise for the angle measurement function. There is a precise statement of this key result in Theorem $\mathbf{V.5.4}$ of the course notes, and references for a proof are given after the statement of the theorem. The basic idea of the argument is to construct a Cartesian coordinate system in the abstract plane **P** or space **S**.

Passage from the analytic to the synthetic approach

(excluding areas and volumes)

This requires purely algebraic and analytic verifications that the analytically defined data satisfy the axioms in the first seven groups of axioms for synthetic geometry. One way of approaching this is outlined in http://math.ucr.edu/~res/math133/verifications.pdf. The idea is to replace the axioms of this course with shorter lists of data and axioms for Euclidean geometry, and to verify that the analytically defined system satisfies the shorter list of conditions. A few steps in this process are highly nonelementary.