

# ADDITIONAL FILES FOR UNIT V

## Basic material (may be covered on quizzes or examinations)

<http://math.ucr.edu/~res/math133/geometrynotes05a.f13.pdf>

<http://math.ucr.edu/~res/math133/geometrynotes05b.f13.pdf>

<http://math.ucr.edu/~res/math133/geometrynotes05c.f13.pdf>

The course notes for this unit (not really additional files).

<http://math.ucr.edu/~res/math133/neutral-proofs1.pdf>

<http://math.ucr.edu/~res/math133/neutral-proofs2.pdf>

<http://math.ucr.edu/~res/math133/neutral-proofs3.pdf>

These files provide detailed information about proofs for certain basic geometric theorems in the framework of neutral geometry. **The first document is absolutely central to the course**, and it gives synthetic, neutral geometric, proofs of all the earlier theorems listed in Section **V.2** of the course notes; in some cases the earlier proofs go through in neutral geometry, and new proofs are given when the earlier proofs are not valid in that setting.

The remaining two documents prove other results which are valid in neutral geometry but are not needed in Unit **V** of the course notes. In particular, the second document shows that the theorem on triangle incenters from Section **III.4** in the course notes is also valid in neutral geometry, and the third document, which is written at a substantially higher level than most of the material in this course, proves the results on intersecting lines and circles from Section **III.6** in the course notes in neutral geometry. None of the material in the second or third file will be covered on the third examination.

<http://math.ucr.edu/~res/math133/neutral-rectangles.pdf>

This file gives details for the proof of Theorem **V.3.8** that was sketched in the course notes. The exposition and arguments are at the level of the course notes.

<http://math.ucr.edu/~res/math133/neutral-Aristotle.pdf>

This file proves one result in Euclidean geometry which is logically equivalent to Euclid's Fifth Postulate. One way of paraphrasing the result is that two coplanar lines are not asymptotic at infinity in either direction.

<http://math.ucr.edu/~res/math133/pseudosphere.pdf>

This file discusses a surface of revolution called the **pseudosphere**, whose geometry is closely related to hyperbolic geometry; specifically, if one cuts this surface of revolution open along the curve which is rotated about an axis, then the cut open surface is geometrically equivalent to a portion of the hyperbolic plane (which is sketched in the file).

<http://math.ucr.edu/~res/math133/cartoon-physics.pdf>

This file contains lighthearted comments and references to illustrate that one can imagine a world with alternative laws of physics; in contrast, it is difficult or impossible to imagine similar alternatives to the basic laws of arithmetic.

<http://math.ucr.edu/~res/math133/threedim-angles.pdf>

This file, which was also cited for Unit **III**, gives some basic results on **3** – dimensional solid angles, with several exercises and their solutions. Its relevance to the present unit is that the results have implications for the appropriate notion of triangles in spherical geometry.

<http://math.ucr.edu/~res/math133/aabUpdate11f13.pdf>

This file includes lists of the assigned exercises for this unit of the course.

<http://math.ucr.edu/~res/math133/math133exercises05.f13.pdf>

The entire set of exercises for this unit of the course.

<http://math.ucr.edu/~res/math133/math133solutions07.f13.pdf>

<http://math.ucr.edu/~res/math133/math133solutions07.figures.f13.pdf>

<http://math.ucr.edu/~res/math133/math133solutions08.f13.pdf>

These files contain solutions to exercises for this unit of the course and drawings to accompany the first file of solutions as indicated.

<http://math.ucr.edu/~res/math133/crossrefsStillwell.pdf>

Several comments in this file about Unit **V** contain information not repeated elsewhere.

## **Supplementary material**

As before, many files in <http://math.ucr.edu/math133/geometrynotes01x.f13.pdf> under the heading, “Supplementary material,” could be added to this list, and the same applies to the first four files listed in <http://math.ucr.edu/math133/geometrynotes02x.f13.pdf> and all the files in <http://math.ucr.edu/math133/geometrynotes03x.f13.pdf> under the heading, “Supplementary material.”

<http://math.ucr.edu/~res/math133/metgeom.pdf>

This file was also mentioned in connection with Units **II** and **III**; it is also cited here because it gives analytic proofs of some basic results about the standard Cartesian model for the axioms of Euclidean geometry.

<http://math.ucr.edu/~res/math133/nonmetric-models.pdf>

This document considers more classical approaches to describing axioms for synthetic geometry, without the explicit use of the real number system as in the course notes.

<http://math.ucr.edu/~res/math133/synthetic-3Daffine.pdf>

This file proves a few elementary theorems about **3** – dimensional geometrical systems which satisfy the Incidence Axioms and Playfair’s Postulate.

<http://math.ucr.edu/~res/math133/irreducibleplanes1.pdf>

<http://math.ucr.edu/~res/math133/irreducibleplanes2.pdf>

These files discuss proofs that certain abstract incidence planes (for example, those which satisfy some standard conditions like Playfair's Postulate or have a notion of betweenness) are irreducible in the sense of the definition on page **11** in the file

<http://math.ucr.edu/~res/math133/nonmetric-models.pdf>.

<http://math.ucr.edu/~res/math133/axioms4geometry.pdf>

This file describes very short but complete lists axioms for synthetic geometry; Euclidean, neutral and hyperbolic geometry are considered in both **2** and **3** dimensions.

<http://math.ucr.edu/~res/math133/verifications.pdf>

This file was also mentioned in connection with Unit **II**; it discusses the arguments required to verify that the standard models for Euclidean and hyperbolic geometry satisfy one version of the synthetic axioms for these objects. The proofs are self – contained except for a reference to <http://math.ucr.edu/~res/math133/trianglecongruence.pdf> at one step.

<http://math.ucr.edu/~res/math133/synthetic-3Daffine.pdf>

This file proves a few theorems about **3** – dimensional geometrical systems which satisfy the Incidence Axioms and Playfair's Postulate.

<http://math.ucr.edu/~res/math133/skew-lines.pdf>

This file proves a basic result about noncoplanar lines in Euclidean geometry; namely, they have a unique common perpendicular, and the closest points on the original two lines are the points which lie on this perpendicular. The proof uses vector geometry.

## **Optional material**

As before, the files listed in each of the previously cited documents

<http://math.ucr.edu/math133/geometrynotes01x.f13.pdf>

<http://math.ucr.edu/math133/geometrynotes02x.f13.pdf>

<http://math.ucr.edu/math133/geometrynotes03x.f13.pdf>

under the heading, "Optional material," could be added to this list.

<http://math.ucr.edu/~res/math133/lobachevsky-song.pdf>

This file contains commentary on a satirical song which uses Lobachevsky's name.

<http://math.ucr.edu/~res/math133/zzz-math-headlines.pdf>

<http://math.ucr.edu/~res/math133/zzz-poincare-trivia.pdf>

The first of these files contains a newspaper – like story about John F. Nash, who is mentioned in Section **V.6** of the course notes (and was the subject of an academy award winning film), and the second describes the relationship between *Henri Poincaré* (who is mentioned in the course notes repeatedly) and *Raymond Poincaré*, who was a prominent French political leader during the first three decades of the 20<sup>th</sup> century.