Logic Exercises

These exercises correspond to virtual Section **I.0** from the course notes (Background material). They are taken from pages 8–10 (Section 1.1, *Inductive and Deductive Reasoning*) of the following book:

> **M. L. Lial. B. A. Brown, A. R. Steffensen, and L. M. Johnson.** *Essential of Geometry for College Students* (Second Edition). Pearson Addison-Wesley, Boston, 2004. **ISBN:** 0-201-74882-7

The files `braintest*.pdf`, where $i = 1, 2, 3, 4$, contain additional logical problems, some of which resemble those given below.

43. The number of marbles in a jar doubles every minute, and the jar is filled after 10 minutes. When is it half full?

44. An inflatable raft will carry at maximum 200 pounds. How can a man weighing 200 pounds and two children weighing 100 pounds each use the raft to reach an island?

46. A judge wishing to convict a defendant puts two pieces of paper into a hat. He tells the jury that if the defendant draws the piece of paper marked "guilty" he will be convicted, but if he draws the piece marked "innocent" he will be set free. The hitch is that the judge wrote "guilty" on both pieces of paper. But when the crafty defendant showed the jury one piece of paper, the judge was forced to let him go free. How did the defendant outwit the judge?

47. You have three pouches, each of which contains three coins. Two contain real coins which weight 1 lb. each , and the third contains counterfeit coins, each of which weighs 1 lb. 1 oz. A scale is available, but it can be used only once to obtain a particular measure of weight. How might you use the scale to determine which pouch contains the counterfeit coins?

54. Consider the following set of six "postulates:"

(a) Students A, B, C, D, E are students at the same institution where one is a freshman, one is a sophomore, one is a junior, one is a senior, and one is a graduate student.

(b) A, B, and C have not completed their undergraduate work.

(c) B is one year ahead of E.

(d) A is not a freshman.

(e) E is not a freshman.

(f) A is more advanced than E.

Determine which student belongs to which class. Justify your conclusion formally with a sequence of theorems about the axiomatic system.

55. Consider the following eight "postulates:"

(a) Smith, Jones and Brown are the pilot, copilot and flight attendant on an airplane, but not necessarily in that order.

(b) There are three passengers on the train with the same names.

(c) The flight attendant lives in Denver.

(d) Passenger Brown lives in San Francisco.

(e) Passenger Jones long ago forgot all the algebra he learned in high school.

(f) The passenger with the same name as the brakeman lives in New York.

(g) The flight attendant and one of the passengers, a professor of mathematical physics, attend the same health club.

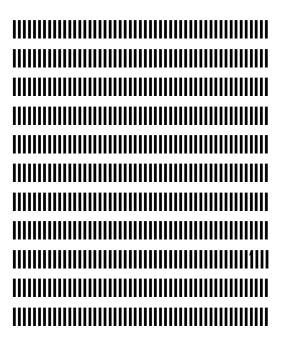(h) Smith beat the copilot in a game of tennis near their homes.

Determine the names of the pilot, copilot and flight attendant. Justify your conclusion formally with a sequence of theorems about the axiomatic system.

Letter Searches

Looking through things carefully and thoroughly is an important part of creating and debugging mathematical arguments. Here are a few simple exercises which test the ability to locate something which seems exceptional or out of place.

Find the B's (there are two of them)

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MORE EXERCISES ON BARYCENTRIC COORDINATES

Here are some more computational exercises involving barycentric coordinates for Sections **I.4** and **I.5** of the course notes. They are included as additional practice in working such problems (which are fundamental to the course).

A1.   Let $A = (-1, 1)$, $B = (1, 1)$ and $C = (1, -1)$. Find the barycentric coordinates of D with respect to A, B, C if (1) $D = (a, a)$ where a is some scalar, (2) $D = (\frac{2}{3}, \frac{3}{4})$.

A2.   Let $A = (1, 0)$, $B = (-1, -1)$ and $C = (0, 1)$. Find the barycentric coordinates of D with respect to A, B, C if $D = (n, 10 - n)$ where n is an integer and $0 \leq n \leq 5$.

A3.   Let $A = (0, 2)$, $B = (1, 0)$ and $C = (3, 0)$. Find the barycentric coordinates of D with respect to A, B, C if $D = (1, x)$ where x is equal to $\frac{1}{2}$, 1, 2 and 4.