

## Solutions to Logic Exercises

**43.** The jar will be full in 9 minutes. We can restate the condition to say that if  $A$  is the amount at time  $T$  minutes, then  $A/2$  is the amount at time  $T - 1$  minutes.

**44.**  $A$  is the man,  $B$  and  $C$  are the two children. Here is a list of where everyone is, starting with everyone on the mainland; each step represents a raft crossing.

- (i) All are on the mainland, and none are on the island.
  - (ii)  $B$  and  $C$  take the raft from the mainland to the island.
  - (iii) Now  $A$  is on the mainland, and  $B$  and  $C$  are on the island.
  - (iv)  $B$  takes the raft from the island to the mainland.
  - (v) Now  $A$  and  $B$  are on the mainland, and  $C$  is on the island.
  - (vi)  $A$  takes the raft from the mainland from the island.
  - (vii) Now  $B$  is on the mainland, and  $C$  and  $A$  are on the island.
  - (viii)  $C$  takes the raft from the island to the mainland.
  - (ix) Now  $B$  and  $C$  are on the mainland, and  $A$  is on the island.
  - (x)  $B$  and  $C$  take the raft from the mainland to the island.
- Now none are on the mainland, and all are on the island.■

**46.** The defendant produced a written confession from someone else.

**47.** Identify the pouches as  $A$ ,  $B$ ,  $C$ . Take one coin from  $A$ , two from  $B$  and 3 from  $C$ . The weight will be 6 lb. 1 oz., 6 lb. 2 oz., or 6 lb. 3 oz., depending upon which of the pouches contains counterfeit coins, and the number of counterfeit coins will be the number of "extra" ounces.■

**54.** Let  $n(X)$  be the year of study for each student. Then  $n(C) = 1$ ,  $n(E) = 2$ ,  $n(B) = 3$ ,  $n(A) = 4$ , and  $n(D) = 5$ . Here is the derivation:

- (i) The assumptions imply that  $n(B) = n(E) + 1$ ,  $n(A), n(E) > 1$ ,  $n(A), n(B), n(C) \leq 4$ , and  $n(A) > n(E)$ .
- (ii) The first and third conditions imply that  $n(B) > 2$ .
- (iii) The second and fourth conditions imply that  $n(A) > 2$ .
- (iv) The first condition implies that  $n(E) \leq 4$ .
- (v) The previous statement and the third condition imply that  $n(D) = 5$ .
- (vi) Therefore  $3 \leq n(A), n(B) \leq 4$ .
- (vii) Therefore  $1 \leq n(C), n(E) \leq 2$ .
- (viii) By the preceding statement and the second condition, we must have  $n(E) = 2$ , so that  $n(C) = 1$ .
- (ix) By the preceding statement and the first condition, we must have  $n(B) = 3$ , so that  $n(A) = 4$ .■

**55.** The pilot is Smith, the flight attendant is Jones, and the copilot is Brown. Passenger Smith lives in Denver, passenger Jones lives in New York, and passenger Brown lives in San Francisco. Here is the derivation:

Let  $S, J, B$  denote the flight crew, and let  $S', J', B'$  be the corresponding passengers. Define a namesake function by  $n(X) = X'$ ; for each member of the flight crew, this function yields the passenger with the same name.

Let  $t$  be the title function for the flight crew, so that the values are  $P, C, A$ . There is also a home function  $h$  whose values may include  $SF, D$  and  $NY$ . Also, there is a vocation function  $v$  for the passengers, and its values include  $MP$ .

In these terms, here is what we are given:

$$(c') \quad h(A) = D$$

$$(d') \quad h(B') = SF$$

$$(e') \quad v(J') \neq MP$$

$$(f') \quad t(A) = X \text{ implies } h(X') = NY$$

$$(g') \quad h(A) = h(Y') \text{ if } v(Y') = MP$$

$$(h') \quad h(S) = h \circ t^{-1}(C).$$

By  $(c')$  and  $(f')$  we have  $X' \neq B'$ , so that  $X \neq B$ .

By  $(f')$  we now have  $B \neq t^{-1}(A)$ .

Therefore  $X = S$  or  $J$ , and  $t(B) = P$  or  $C$ .

By  $(d')$ ,  $(e')$ ,  $(f')$  and  $(g')$ ,  $h(A) = D = h(S')$  or  $h(B')$ .

By  $(d')$ ,  $D = h(S')$ .

By  $(f')$  and the previous steps,  $h(J') = NY$ .

By  $(f')$ ,  $t(J) = A$ .

Therefore, we have either  $t(S) = P$  and  $t(B) = C$  or vice versa.

By  $(h')$ , we have  $S \neq t^{-1}(C)$ .

Summarizing, we have the following:

$$t(S) = P, \quad t(B) = C, \quad t(J) = A$$

$$h(S') = D, \quad h(J') = NY, \quad h(B') = SF \quad \blacksquare$$





