Solutions to L1 – L4

Note that there are many other valid choices for $\vec{p}$ and $\{\vec{v}_1, \dots, \vec{v}_k\}$ in each problem.

L1. Write out $(A : b)$ such that $Ax = b$.

$$\begin{pmatrix} 1 & -3 & 5 & : & 3 \end{pmatrix}$$

This is already in row-reduced echelon form. y + z can be arbitrary, but x is determined by them uniquely.

Particular Solution: Set $y = z = 0$

$$\vec{p} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}.$$

Reduced Equation: $(A : 0) = \begin{pmatrix} 1 & -3 & 5 & : & 0 \end{pmatrix}$

Basis given by $\vec{v}_1$ with $y = 1$ + $z = 0$
$\vec{v}_2$ with $y = 0$ + $z = 1$.

So $\vec{v}_1 = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix}$.

General solution: $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + t_1 \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix}$.

L2. Use the same method. $(A:b)$ is now

$(2 \;\; 4 \;\; -1 : 5)$ with row reduced echelon form

$(1 \;\; 2 \;\; -\frac{1}{2} : 5/2)$. $\vec{p}$ has $y = z = 0$, so

$\vec{p} = \begin{pmatrix} 5/2 \\ 0 \\ 0 \end{pmatrix}$. The reduced eqn is

$$2x + 4y - z = 0$$
$$x + 2y - \tfrac{1}{2}z = 0,$$

and the corresponding solutions are

$\vec{V}_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ $\vec{V}_2 = \begin{pmatrix} \frac{1}{2} \\ 0 \\ 1 \end{pmatrix}$ and the general soln. is

$$\begin{pmatrix} 5/2 \\ 0 \\ 0 \end{pmatrix} + t_1 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix}.$$

L3. Now $(A:b) = \begin{pmatrix} 1 & -1 & 1 & : 4 \\ 2 & 1 & 3 & : 10 \end{pmatrix}$.

Put in echelon form.

$$\begin{pmatrix} 1 & -1 & 1 & : 4 \\ 2 & 1 & 3 & : 10 \end{pmatrix} \xrightarrow[2\times 1\text{st}]{2\text{nd } \text{minus}} \begin{pmatrix} 1 & -1 & 1 & : 4 \\ 0 & 3 & 1 & : 2 \end{pmatrix}$$

$$\xrightarrow[2\text{nd}]{\frac{1}{3}\times} \begin{pmatrix} 1 & -1 & 1 & : 4 \\ 0 & 1 & \frac{1}{3} & : 2/3 \end{pmatrix} \xrightarrow[\substack{(-1)\times 2\text{nd} \\ = 1\text{st} + 2\text{nd}}]{1\text{st minus}} \begin{pmatrix} 1 & 0 & 4/3 & : 14/3 \\ 0 & 1 & 1/3 & : 2/3 \end{pmatrix}$$

z arbitrary, x & y depend on z.
(1-dim solution space for reduced eqn.)

Particular solution: Set $z=0$, solve for $x + y$. Get $\vec{p} = \begin{pmatrix} 14/3 \\ 2/3 \\ 0 \end{pmatrix}$

Reduced equations: System is

$$\left(\begin{array}{ccc:c} 1 & 0 & 4/3 & 0 \\ 0 & 1 & 1/3 & 0 \end{array}\right)$$

and has a 1-dim solution space. Get a basis vector by setting $z=1$, solving for $x + y$: $\vec{v}_1 = \begin{pmatrix} -4/3 \\ -1/3 \\ 1 \end{pmatrix}$

General solution is $\begin{pmatrix} 14/3 \\ 2/3 \\ 0 \end{pmatrix} + t_1 \begin{pmatrix} -4/3 \\ -1/3 \\ 1 \end{pmatrix}$

L4. Same idea as in L3.

$$(A:b) = \left(\begin{array}{ccc:c} 1 & 4 & -3 & 2 \\ 1 & -2 & 2 & 1 \end{array}\right) \xrightarrow[\text{1st}]{\text{2nd minus}}$$

$$\left(\begin{array}{ccc:c} 1 & 4 & -3 & 2 \\ 0 & -6 & 5 & -1 \end{array}\right) \xrightarrow[\text{2nd}]{-1/6\ \times} \left(\begin{array}{ccc:c} 1 & 4 & -3 & 2 \\ 0 & 1 & -5/6 & 1/6 \end{array}\right)$$

$$\xrightarrow[4\times\text{2nd}]{\text{1st} -} \left(\begin{array}{ccc:c} 1 & 0 & 1/3 & 4/3 \\ 0 & 1 & -5/6 & 1/6 \end{array}\right)$$

To find $\vec{p}$, set $z=0$ + solve for x & y:

$$\vec{p} = \begin{pmatrix} 4/3 \\ 1/6 \\ 0 \end{pmatrix}.$$

Reduced system is then

$$\left(\begin{array}{ccc:c} 1 & 0 & 1/3 & 0 \\ 0 & 1 & -5/6 & 0 \end{array}\right)$$

& has a 1-dim solution space. To find $\vec{V}_1$, set $z=1$ & solve for x & y.

$$v_1 = \begin{pmatrix} -1/3 \\ 5/6 \\ 1 \end{pmatrix}$$

and the general soln is

$$\begin{pmatrix} 4/3 \\ 1/6 \\ 0 \end{pmatrix} + t_1 \begin{pmatrix} -1/3 \\ 5/6 \\ 1 \end{pmatrix}.$$