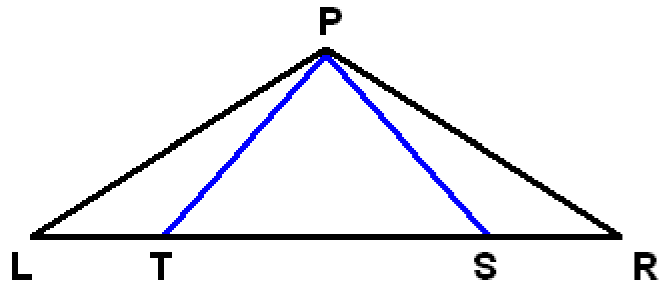
FIGURES FOR SOLUTIONS TO SELECTED EXERCISES

III : Basic Euclidean Concepts and theorems

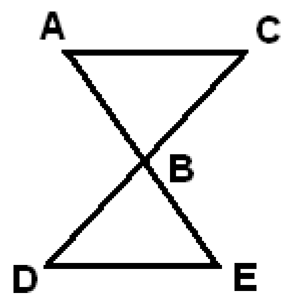
III.2 : Basic theorems on triangles

III.2.3.



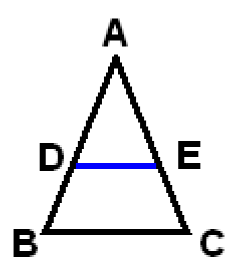
We are given that **R∗S∗T,** d**(R, S)** = d**(L, T),** and d**(P, S)** = d**(P, T).** The objective is to prove that we have overlapping congruent triangles △**RTP** ≅ △**LSP** and that we also have |∠**PSR**| = |∠**PTL**|.

III.2.4.



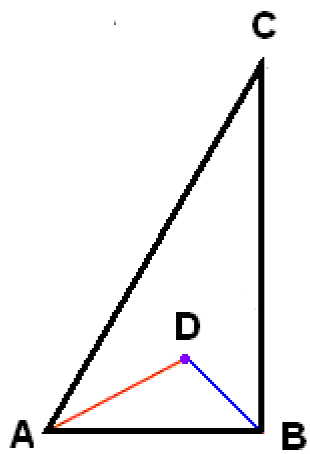
The point **B** which is the midpoint of both **[AE]** and **[CD],** and the objective is to prove that **AC** || **DE.** One method for doing this is to find a pair of alternate interior angles**.**

III.2.6.



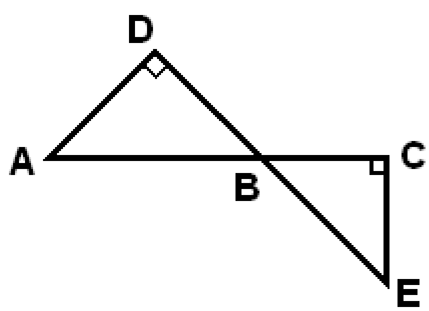
We are given that d(**A, B**) = d(**A, C**), and that **D** and **E** are points of **(AB)** and **(AC)** such that d(**A, D**) = d(**A, E**). To prove that **BC** || **DE,** it suffices to find a pair of corresponding angles.

III.2.7.



More generally, if **D** is a point in the interior of △**ABC** such that **[AD** bisects ∠**CAB** and **[BD** bisects ∠**CBA,** then there is a formula relating |∠**ADB**| and |∠**ACB**|.

III.2.8.



The perpendicular pairs of lines are marked in the drawing, and the objective is to show that |∠**DAB**| = |∠**BEC**|.

Note*:** This exercise is actually a special case of a result stated in many high school geometry texts: ***If two angles have their corresponding sides perpendicular, left to left and right to right, then the angles have equal measurements. — More formally, the general result can be stated as follows: ***If*** ∠**DAB** ***and*** ∠**FEC** ***are such that*** **AB** ⊥ **EC** ***and*** **AD** ⊥ **EF*, then*** |∠**DAB**| = |∠**FEC**|. — In order to prove the general result, it is necessary to look at the points **X, Y, Z** and **W** where (respectively) **AB** meets **EC, AD** meets **EF, AD** meets **EC,** and **AB** meets **EF** and find where they lie on the appropriate lines; Lemma **III.4.2** guarantees that the last two pairs of lines have points in common. For the case studied in the exercise, the point **X** (which is just **C**) lies on **(AB** and **(EC,** the point **Y** (which is just **D**) lies on **(AD** and **(EF,** the point **W** (which is just **B**) lies on **(AB** and **(EF,** and the drawing suggests that the point **Z** lies on **(AD** and **(EC.** It is also necessary to analyze all the possible betweenness conditions which can hold for the various triples of collinear points.