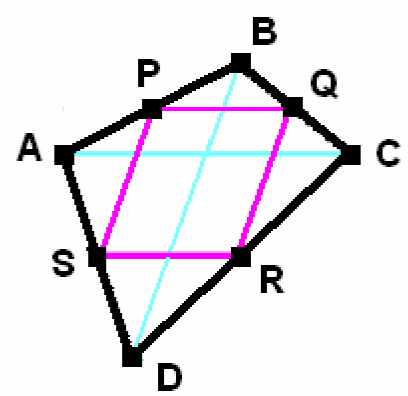
FIGURES FOR SOLUTIONS TO SELECTED EXERCISES

III : Basic Euclidean Concepts and theorems

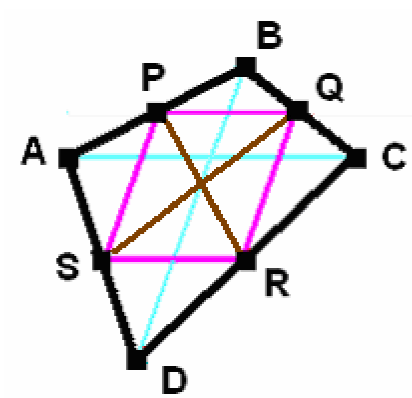
III.3 : Convex polygons

III.3.1.



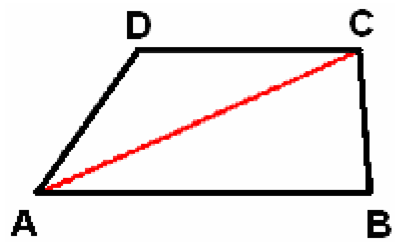
In this figure, each of the diagonals of the convex quadrilateral determines a pair of triangles which share a common edge. The midpoints of the sides of the original convex quadrilateral are also midpoints of the sides of these triangles.

II.3.2.



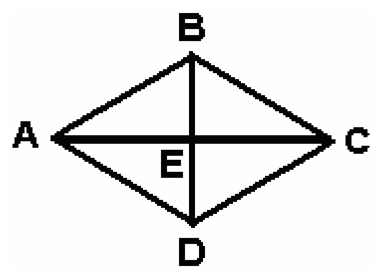
By a previous exercise we know that the midpoints **P, Q, R, S** (in that order) determine a parallelogram.

II.3.4.



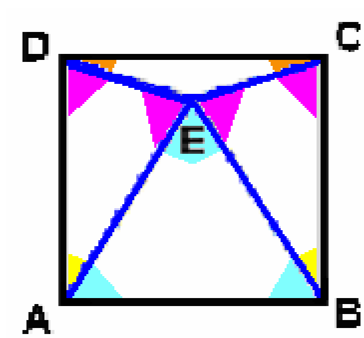
We are given that d(**A, D**) = d(**C, D**) in the trapezoid illustrated above, and the objective is to show that **[AC** bisects $\angle$**DAB.** Note that $\triangle$**DAC** is isosceles.

III.3.6.



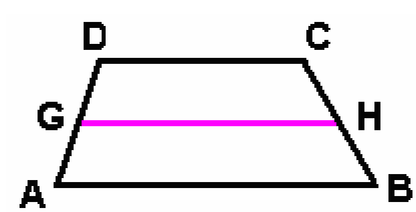
The points **A, B, C, D** form the vertices of a parallelogram, and the objective is to show that this parallelogram is a rhombus (all sides have equal length) if and only if **AC** is perpendicular to **BD.**

II.3.7.

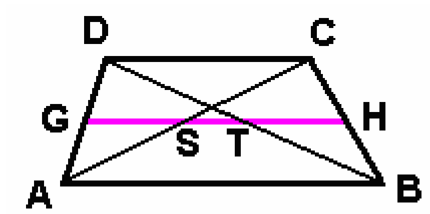


In the drawing, two angles have equal measurement if they are marked with the same color.

III.3.14.

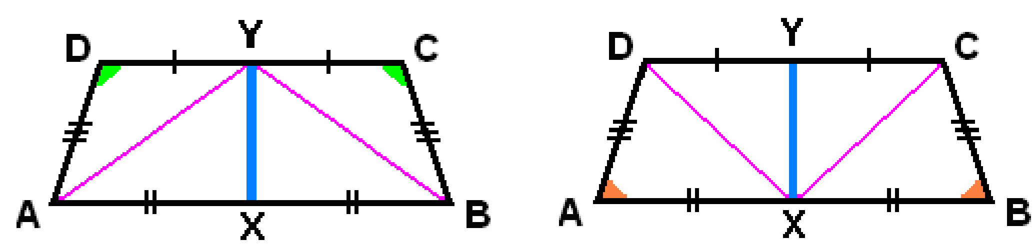


The points **A, B, C, D** form the vertices of a trapezoid with **AB** || **CD,** and the midpoints of **[AD]** and **[BC]** are **G** and **H** respectively**.** In the first part of this exercise**,** the objective is to show that **GH** is parallel to **AB** and **CD** and to find its length in terms of the lengths of **[AB]** and **[CD].**



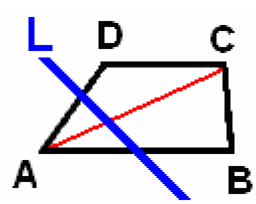
In the final part of the exercise the objective is to show that the midpoints **S** and **T** of the diagonals **[AC]** and **[BD]** both lie on the line **GH.**

III.3.15.



We are assuming that the points **A, B, C, D** form the vertices of an ***isosceles*** trapezoid with **AB** || **CD,** and the points **X** and **Y** are the midpoints of **[AB]** and **[CD]** respectively**.** The idea is to use the basic properties of an isosceles trapezoid (various parts have equal measurements) to show that $\triangle$**ADY** $\cong$ $\triangle$**BCY** and $\triangle$**XAD** $\cong$ $\triangle$**XBC,** so that **Y** will be equidistant from **A** and **B,** and **X** will be equidistant from **C** and **D.**

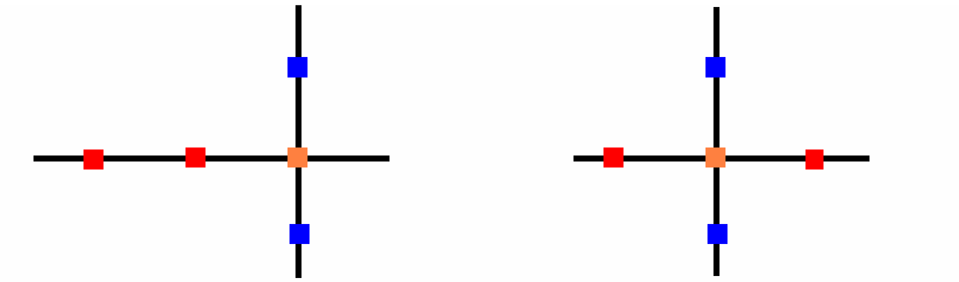
III.3.20.



Split the convex quadrilateral into two triangles along the diagonal **[AC],** and apply Pasch's Theorem successively to $\triangle$**ABC** and $\triangle$**ADC,**

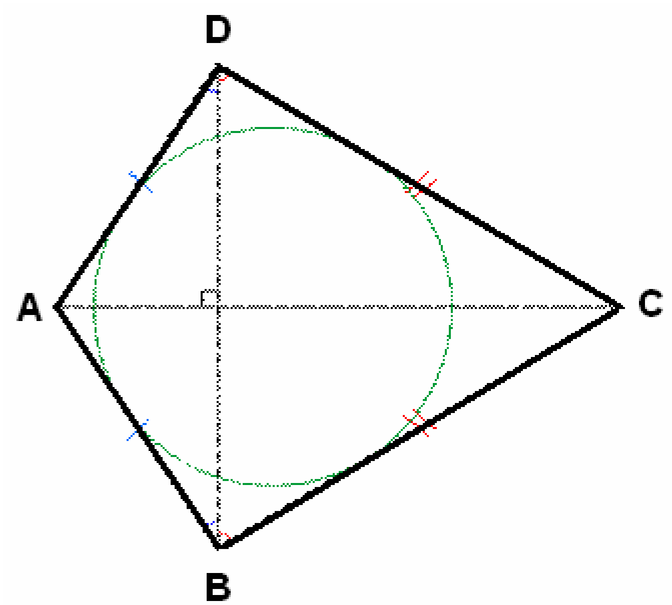
III.3.21.

If $\mathbf{A_3}$ and $\mathbf{A_4}$ lie on the same side of $\mathbf{A_1A_2}$ the conclusion of the exercise is satisfied**,** so assume that this is not the case**.** In the drawing on the next page the points $\mathbf{A_3}$**,** $\mathbf{A_4}$ (the blue dots in the drawing on the next page) lie on opposite sides of the line $\mathbf{A_1A_2}$ (and the points $\mathbf{A_1}$**,** $\mathbf{A_2}$ are the red dots)**.** The point **Y** where the line $\mathbf{A_1A_2}$ meets the open segment $\mathbf{(A_3A_4)}$ is the orange dot**.** There are two cases depending upon whether or not **Y** lies between $\mathbf{A_1}$ and $\mathbf{A_2}$ (the drawing on the left describes the case where this does not hold, and the drawing on the right describes the case where it does). In the drawing we have not labeled A_1, A_2, A_3, A_4 explicitly because the picture does not depend upon which of the red points is labeled A_1 and which is labeled A_2, and likewise the picture does not depend upon which of the blue points is lableled A_3 and which is labeled A_4.



III.3.22.

Here is a drawing of a convex kite as defined in the exercise:



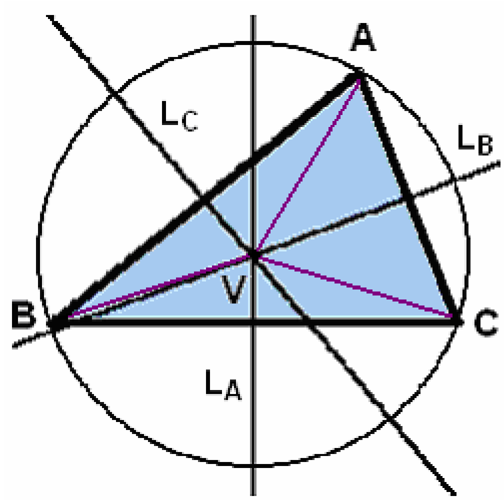
(**Source:** http://en.wikipedia.org/wiki/File:GeometricKite.svg)

The objectives are to show that the vertex angles at **B** and **D** have equal measures and that line **AC** is the perpendicular bisector of the segment **[BD].**

> **Note:** The drawing suggests that it is possible to inscribe a circle inside such a quadrilateral such that each side of the latter is a tangent line to the circle. This is in fact the case, and the proof is described in Exercise **III.6.15.**

III.4 : Concurrence theorems

III.4.2.

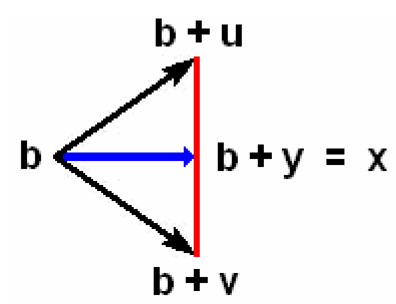


Given that the circumcenter **V** of △**ABC** lies in the interior of that triangle, the goal is to prove that all three vertex angles are acute. By the Addition Postulate for angle measurement, at each vertex **D** the measure of the vertex angle at **D** can be written as a sum of the measurements of two other angles in the drawing; note that the angle sum for each triangle in the drawing is equal to 180 degrees.

III.4.5.

See the online file http://math.ucr.edu/~res/progeom/treasure-problem.pdf for a drawing in which different possibilities for the unknown position are plotted (however, the solution in that document requires material not covered in the notes thus far).

III.4.11.



Assume the notation in the exercise. Then **b**, **b + u** and **b + v** form the vertices of an isosceles triangle such that both edges containing **b** have length 1. The point **x** is the midpoint of the line segment joining **b + u** and **b + v**.