SOLUTIONS TO ADDITIONAL EXERCISES ON BARYCENTRIC COORDINATES

Here are the solutions to the additional exercises **A1.**–**A3** in `update05w09.pdf`. Illustrations to accompany these solutions are given in the online file

`barycentric-figures.pdf`

in the course directory.

A1. If D is the point whose barycentric coordinates are to be found, then we need to express $D - A$ as a linear combination of $B - A$ and $C - A$. The data in the problem imply that $B - A = (2, 0)$ and $C - A = (2, -2)$.

For the first part of the problem we know that $D - A = (a + 1, a - 1)$. Thus we need to solve the system of linear equations given by

$$(a + 1, a - 1) \;=\; v(2, 0) \;+\; w(2, -2) \;=\; (2v + 2w, -2w)$$

for v and w; the third barycentric coordinate is then given by $u = 1 - v - w$. The solution to the displayed system of equations is given by $w = \frac{1}{2}(1 - a)$ and $v = a$. Using these and $u + v + w = 1$, we also conclude that $u = \frac{1}{2}(1 - a)$.■

For the second part or the problem, we have $D = (\frac{2}{3}, \frac{3}{4})$, so that $D - A = (\frac{5}{3}, -\frac{1}{4})$ and the corresponding system of equations is

$$2v \;+\; 2w \;=\; \tfrac{5}{3}\,, \qquad -2w \;=\; -\tfrac{1}{4}\,.$$

The solution to this systen is $v = \frac{17}{24}$ and $w = \frac{1}{8}$, so that $u = \frac{1}{6}$.■

A2. We have $B - A = (-1, -2)$ and $C - A = (1, -1)$. Since $D - A = (n, 9 - n)$ where $0 \leq n \leq 5$, it follows that we need to solve the system

$$(n, 9 - n) \;=\; v(-1, -2) \;+\; w(1, -1) \;=\; (-v + w, -2v - w)$$

for v and w and then to set $u = 1 - v - w$. It follows that $v = -3$, $w = n - 3$ and $u = 7 - n$.

n	u	v	w
0	7	−3	−3
1	6	−3	−2
2	5	−3	−1
3	4	−3	0
4	3	−3	1
5	2	−3	2

A3. We have $B - A = (1, -2)$, $C - A = (3, -2)$, and $D - A = (0, x + 2)$. Therefore we need to solve the system

$$(0, -2) \;=\; v(1, -2) \;+\; w(3, -2) \;=\; (v + 3w, -2v - 2w)$$

for v and w when x takes one of the specified values, and once again we can compute u from v and w. The solutions to the system of equations is $v = 1 - \frac{3}{4}x$, $w = \frac{1}{4}x$, and $u = \frac{1}{2}u$. Here is the corresponding table of values:

x	u	v	w
$1/2$	$1/4$	$5/8$	$1/8$
1	$1/2$	$1/4$	$1/4$
2	1	$-1/2$	$1/2$
4	2	-2	1