Math 133
Winter 2009

MORE EXERCISES FOR SECTIONS II.1 AND II.2

B1. Let L be a line in $\mathbb{R}^3$, and let $\mathbf{x}$ be a point which does not lie on L. Using the Incidence Axioms, prove that there is a unique plane P such that $\mathbf{x} \in P$ and $L \subset P$.

B2. Suppose that we are given three distinct lines L, M, N in $\mathbb{R}^3$ such that (i) the lines all contain some point X, (ii) each of the lines has a point in common with a fourth line K which does not contain X. Prove that there is a plane containing all four lines.

B3. Suppose that we are given points A, B, C and X, Y, Z such that $A * B * C$ and $X * Y * Z$ both hold, and in addition we have $d(A, C) = d(X, Z)$ and $d(B, C) = d(Y, Z)$. Prove that $d(A, B) = d(X, Y)$. [*If equals are subtracted from equals, the differences are equal.*]

B4. Suppose that we are given points A, B, C such that $A * B * C$. Prove that (AB) is a proper subset of (AC).

B5. Suppose that $A \neq B$; if AB is the line joining A to B, prove that $AB = [AB \cup [BA$.

B6. Suppose that $A * B * C$; prove that $[AB = [AB] \cup [BC$.

B7. Let L be a line in $\mathbb{R}^2$, and let M be a second line in $\mathbb{R}^2$ such that L and M meet at the point A.

(a) If X and Y are points of M such that $A * X * Y$ is true, prove that X and Y lie on the same side of L.

(b) If X and Y are points of M such that $X * A * Y$, prove that X and Y lie on opposite sides of L. [*Hint:* For both parts of this problem, show that the alternatives are impossible.]

B8. Suppose that we are given distinct points A, B in $\mathbb{R}^2$, and suppose also that C and D lie on opposite sides of the line AB. Prove that $[AC$ and $[BD$ have no points in common.

B9. (i) Suppose that we are given three noncollinear points A, B, C in $\mathbb{R}^2$. Prove that $\Delta ABC \cap AB = [AB]$. [*Hint:* If X is a point on $[BC]$ or $[AC]$ other than A or B, explain why X cannot lie on AB.]

B10. For each of the choices below, determine whether X and Y lie on the same side as the line L defined by the corresponding equation.

(a) $X = (3, 6)$, $Y = (1, 7)$, and L is defined by the equation $9x - 4y = 7$.

(b) $X = (8, 5)$, $Y = (-2, 4)$, and L is defined by the equation $y = 3x - 7$.

(c) $X = (7, -6)$, $Y = (4, -8)$, and L is defined by the equation $2x + 3y + 5 = 0$.

(d) $X = (0, 1)$, $Y = (-2, 6)$, and L is defined by the equation $3y = 2 - 7x$.