Mathematics 133, Winter 2009, Examination 1

Answer Key

1. [*20 points*] If L is the line of intersection for the planes defined by $x + y + z = 3$ and $x = 1$, then $(1, 1, 1)$ lies on L. Find a second point on L. [*Hint:* What do you get if you subtract the second equation from the first?]

SOLUTION.

If we subtract the second equation from the first we obtain the equation $y + z = 2$. Therefore the points on the line of intersection all have the form $(1, y, 2 - y)$ where $y \neq 1$, and any such point is a valid answer.

2. [*25 points*] Find the barycentric coordinates of $(3, 2)$ with respect to the three noncollinear points $(1, 0)$, $(5, 0)$ and $(6, 10)$.

SOLUTION.

Let $D = (3, 2)$, and let the remaining points be A, B, C in that order. Then $D - A = (2, 2)$, while $B - A = (4, 0)$ and $C - A = (5, 10)$. The barycentric coordinates v and w of B and C are given by the equation

$$(2,2) \;=\; D-A \;=\; v(B-A) \,+\, w(C-A) \;=\; v(4,0) \,+\, w(5,10) \;=\; (4v+5w, 10w)\,.$$

Equating coordinates, we see that $4v + 5w = 2 = 10w$, so that $w = \frac{1}{5}$ and $v = \frac{1}{4}$. The barycentric coordinate u of A is then $1 - v - w = \frac{11}{20}$.

3. [*30 points*] (a) Give a mathematical definition or characterization of the concept, "Point B is between points A and C," where A, B, C lie in the coordinate plane. [Two ways of doing this are in terms of distance or special types of linear combinations.]

(b) Let A, B, C, D be the collinear points with coordinates $(\frac{1}{2}, 0)$, $(1, 0)$, $(2, 0)$, and $(4, 0)$. For which of the points $X = A, B, C, D$ is B between A and X? For which of the points $Y = A, B, C, D$ is C between B and Y?

SOLUTION.

(a) Two acceptable answers are that $B = A + t(B - A)$ where $0 < t < 1$ or $d(A, C) = d(A, B) + d(B, C)$ with A, B, C distinct.

(b) Every point $(x, 0)$ on the x-axis can be written as $\mathbf{0}+x(B-\mathbf{0})$. Since $\frac{1}{2} < 1 < 2 < 4$, it follows that $A * B * Y$ is true if $Y = C$ or D , while $B * C * Y$ is true if $Y = D$ but not if $Y = A$.

4. [*25 points*] Let L be the line defined by the equation $3x + 4y = 50$. Determine which of the points $(1, 1)$, $(2, 4)$, $(3, 9)$, $(4, 16)$ lie on the same side of L as $(0, 0)$.

SOLUTION.

Write the equation of the line as $0 = 3x + 4y - 50 = g(x, y)$. Then $g(0, 0) = -50 < 0$, while $g(1, 1) = -43 < 0$, $g(2, 4) = -28 < 0$, $g(3, 9) = -5 < 0$ and $g(4, 16) = 26 > 0$. Since two points P and Q lie on the same side of the line of the signs of P and Q are the same, it follows that $(0, 0)$ is on the same side of the line as $(1, 1)$, $(2, 4)$, and $(3, 9)$, but $(0, 0)$ and $(4, 16)$ lie on opposite sides of that line.