

**Mathematics 133**

**Geometry**

**Course Notes**

**Winter 2009 Version**

**Department of Mathematics**

**University of California, Riverside**

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Note: “\*” means that there is no corresponding section in the notes, but there is such a section in the exercises for the course.

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- geometrynotes1.pdf, pp. 1–29, contains Sections I.1–I.5.
- geometrynotes2a.pdf, pp. 30–56, contains Sections II.1–II.2.
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