Logic Exercises

These are taken from pages 8–10 (Section 1.1, *Inductive and Deductive Reasoning*) of the following book:

> **M. L. Lial. B. A. Brown, A. R. Steffensen, and L. M. Johnson.** *Essential of Geometry for College Students* (Second Edition). Pearson Addison-Wesley, Boston, 2004. **ISBN:** 0-201-74882-7

Some but not all of the solutions are in that book.

43. The number of marbles in a jar doubles every minute, and the jar is filled after 10 minutes. When is it half full?

44. An inflatable raft will carry at maximum 200 pounds. How can a man weighing 200 pounds and two children weighing 100 pounds each use the raft to reach an island?

46. A judge wishing to convict a defendant puts two pieces of paper into a hat. He tells the jury that if the defendant draws the piece of paper marked "guilty" he will be convicted, but if he draws the piece marked "innocent" he will be set free. The hitch is that the judge wrote "guilty" on both pieces of paper. But when the crafty defendant showed the jury one piece of paper, the judge was forced to let him go free. How did the defendant outwit the judge?

47. You have three pouches, each of which contains three coins. Two contain real coins which weight 1 *lb.* each , and the third contains counterfeit coins, each of which weighs 1 *lb.* 1 *oz.* A scale is available, but it can be used only once to obtain a particular measure of weight. How might you use the scale to determine which pouch contains the counterfeit coins?

54. Consider the following set of six "postulates:"

(a) Students A, B, C, D, E are students at the same institution where one is a freshman, one is a sophomore, one is a junior, one is a senior, and one is a graduate student.

(b) A, B, and C have not completed their undergraduate work.

(c) B is one year ahead of E.

(d) A is not a freshman.

(e) E is not a freshman.

(f) A is more advanced than E.

Determine which student belongs to which class. Justify your conclusion formally with a sequence of theorems about the axiomatic system.

55. Consider the following eight "postulates:"

(a) Smith, Jones and Brown are the pilot, copilot and flight attendant on an airplane, but not necessarily in that order.

(b) There are three passengers on the train with the same names.

(c) The flight attendant lives in Denver.

(d) Passenger Brown lives in San Francisco.

(*e*) Passenger Jones long ago forgot all the algebra he learned in high school.

(*f*) The passenger with the same name as the brakeman lives in New York.

(*g*) The flight attendant and one of the passengers, a professor of mathematical physics, attend the same health club.

(*h*) Smith beat the copilot in a game of tennis near their homes.

Determine the names of the pilot, copilot and flight attendant. Justify your conclusion formally with a sequence of theorems about the axiomatic system.

SOLUTIONS

43. 9 minutes. We can restate the condition to say that if A is the amount at time T minutes, then $A/2$ is the amount at time $T-1$ minutes.

44. A is the man, B and C are the two children. Here is a list of where everyone is, starting with everyone on the mainland; each step represents a raft crossing.

(i) All are on the mainland, and none are on the island.

(ii) B and C take the raft from the mainland to the island.

(iii) Now A is on the mainland, and B and C are on the island.

(iv) B takes the raft from the island to the mainland.

(v) Now A and B are on the mainland, and C is on the island.

(vi) A takes the raft from the mainland from the island.

(vii) Now B is on the mainland, and C and A are on the island.

($viii$) C takes the raft from the island to the mainland.

(ix) Now B and C are on the mainland, and A is on the island.

(x) B and C take the raft from the mainland to the island.

Now none are on the mainland, and all are on the island.■

46. Here is one possibility out of several: The defendant produced a written confession from someone else.■

47. Identify the pouches as A, B, C. Take one coin from A, two from B and 3 from C. The weight will be 6 *lb.* 1 *oz.*, 6 *lb.* 2 *oz.*, or 6 *lb.* 3 *oz.*, depending upon which of the pouches contains counterfeit coins, and the number of counterfeit coins will be the number of "extra" ounces.■

54. Let $n(X)$ be the year of study for each student. Then $n(C)=1$, $n(E)=2$, $n(B)=3$, $n(A)=4$, and $n(D)=5$. Here is the derivation:

(i) The stated assumptions imply that $[a']\ n(B)=n(E)+1$, $[b']\ n(A), n(E)>1$, $[c']\ n(A), n(B), n(C)\leq 4$, and $[d']\ n(A)>n(E)$.

(ii) The first and third conditions imply that $n(B)>2$.

(iii) The second and fourth conditions imply that $n(A)>2$.

(iv) The first condition implies that $n(E)\leq 4$.

(v) The previous statement and the third condition imply that $n(D)=5$.

(vi) Therefore $3\leq n(A), n(B)\leq 4$.

(vii) Therefore $1\leq n(C), n(E)\leq 2$.

($viii$) By the preceding statement and the second condition, we must have $n(E)=2$, so that $n(C)=1$.

(ix) By the preceding statement and the first condition, we must have $n(B)=3$, so hat $n(A)=4$.■

55. The pilot is Smith, the flight attendant is Jones, and the copilot is Brown. Passenger Smith lives in Denver, passenger Jones lives in New York, and passenger Brown lives in San Francisco. Here is the derivation:

Let S, J, B denote the flight crew, and let S', J', B' be the corresponding passengers. Define a namesake function by $n(X) = X'$; for each member of the flight crew, this function yields the passenger with the same name.

Let t be the title function for the flight crew, so that the values are P, C, A. There is also a home function h whose values may include SF, D and NY. Also, there is a vocation function v for the passengers, and its values include MP.

In these terms, here is what we are given:

(c') $h(A) = D$

(d') $h(B') = SF$

(e') $v(J') \neq MP$

(f') $t(A) =$X implies $h(X') = NY$

(g') $h(A) = h(Y')$ if $v(Y') = MP$

(h') $h(S) = h \circ t^{-1}(C)$.

We shall use these formulations in the argument given below.

By (c') and (f') we have $X' \neq B'$, so that $X \neq B$.

By (f') we now have $B \neq t^{-1}(A)$.

Therefore $X = S$ or J, and $t(B) = P$ or C.

By (d'), (e'), (f') and (g'), $h(A) = D = h(S')$ or $h(B')$.

By (d'), $D = h(S')$.

By (f') and the previous steps, $h(J') = NY$.

By (f'), $t(J) = A$.

Therefore, we have either $t(S) = P$ and $t(B) = C$ or vice versa.

By (h'), we have $S \neq t^{-1}(C)$.

Summarizing, we have the following:

$t(S) = P$, $t(B) = C$, $t(J) = A$

$h(S') = D$, $h(J') = NY$, $h(B') = SF$ ∎