Math 133
Winter 2009

MORE EXERCISES ON POLYHEDRAL ANGLES

These are numerical exercises involving the fundamental inequalities for a trihedral angle.

E1. Determine whether a trihedral angle can have face angles with the following angle measures, and give reasons for your answers.

(a) 80° , 110° , 140°

(b) 72° , 128° , 156°

(c) 45° , 45° , 90°

(d) 60° , 60° , 60°

(e) 140° , 170° , 171°

(f) 105° , 118° , 130°

E2. A trihedral angle has two face angles whose measures are 80° and 120° respectively. Which of the following values can be the measure of the third face angle? Give reasons for your answer.

$$20^\circ\ , \quad 40^\circ\ , \quad 80^\circ\ , \quad 90^\circ\ , \quad 160^\circ\ , \quad 170^\circ$$

Solutions are given on the next page.

SOLUTIONS.

E1. Each part is answered separately.

(a) Yes, because the largest angle measurement is less than the sum of the smaller two and the sum of all three angle measurements is less than $360°$.■

(b) Yes, for the same reasons as in (a).■

(c) No, because the sum of the smaller two measurements is equal to the largest measurement.■

(d) Yes, for the same reasons as in (a).■

(e) No, because the sum of all three angle measurements is greater than $360°$.■

(f) Yes, for the same reasons as in (a).■

E2. For the first three choices of the angle measure θ we have $\theta \leq 80° < 120°$, and therefore we must also have $120 < 80 + \theta$ and $200 + \theta < 360$. These imply that if $\theta \leq 80°$, then we must also have $\theta > 40°$. This means that $20°$ and $40°$ cannot be realized but $80°$ can. If $\theta = 90°$, then we have $80 \leq \theta \leq 120$ so the conditions for a trihedral angle are still $120 < 80 + \theta$ and $200 + \theta < 360$. Both of these hold if $\theta = 90$, so this value can also be realized. Finally, in the last two cases we have $80 < 120 < \theta$, and since $\theta < 180 < 120 + 80 = 200$, the Triangle Inequality condition is satisfied. However, we also have

$$\theta \ + \ 120 \ + \ 80 \quad \geq \quad 160 \ + \ 120 \ + \ 80 \quad = \quad 360$$

and therefore the second condition for realization is not met.

Summarizing, we know that the only the middle two possibilities can be realized; the first two are eliminated by the Triangle Inequality for trihedral angles, while the last two are eliminated by the constraint that the angle sum is less than $360°$.■