Math 133
Winter 2009

MORE EXERCISES FOR SECTIONS II.3 AND II.4

All of the supplementary exercises for Section I.4 and many of those for Sections II.1–II.2 were fairly elementary and intended to help reinforce basic concepts. In contrast, the exercises below are more challenging and included to cover some additional material.

C1. Suppose that A, B, C are noncollinear points in a plane P, and let $X \in \Delta ABC$. Prove that if X is a vertex of ΔABC then there are (at least) two lines L and M such that X lies on both and each contains at least three distinct points of ΔABC, but if X is not a vertex then there is only one line L in P such that $X \in L$ and L contains at least three points of ΔABC. [*Hint:* The conclusion in Exercise II.2.8 is useful for establishing part of this result.]

C2. Suppose that we are given ΔABC and ΔDEF in a plane P such that $\Delta ABC = \Delta DEF$. Prove that $\{A, B, C\} = \{D, E, F\}$. [*Hint:* Use the preceding exercise.]

C3. Suppose that we are given a triangle ΔABC in a plane P, and suppose that L is a line in P such that L contains a point X in the interior of ΔABC. Prove that L and ΔABC have (at least) two points in common.

C4. [*In this exercise we shall view points of* $\mathbb{R}^n$ *as* $n \times 1$ *column vectors and identify scalars with* 1×1 *matrices in the obvious fashion.*] Let T be an affine transformation of $\mathbb{R}^3$ and write it as $T(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$, where A is an invertible 3×3 matrix and $\mathbf{b}$ is some vector in $\mathbb{R}^3$. Let P be the plane defined by the equation $C\mathbf{x} = d$, where C is a 3×1 matrix and $d \in \mathbb{R}$, and let Q be the image of P; in other words, Q is the set of all vectors $\mathbf{y}$ such that $\mathbf{y} = T(\mathbf{x})$ for some $\mathbf{x} \in P$. Prove that Q is also a plane, and give an explicit equation of the form $U\mathbf{y} = v$ (where U is 1×3 and v is a scalar) which defines Q. [*Hint:* Solve $T(\mathbf{x}) = \mathbf{y}$ for $\mathbf{x}$ in terms of $\mathbf{y}$, A and $\mathbf{b}$.]

NOTE. One specific consequence of the preceding result is that planar figures (which are contained in some plane) and nonplanar figures (which are not contained in any plane) cannot be congruent to each other.

C5. Suppose that we are given coplanar points A, B, C, D such that no three are collinear, and suppose further that the following conditions hold:

(1) B does not lie in the interior of $\angle CAD$.

(2) C does not lie in the interior of $\angle BAD$.

(3) D does not lie in the interior of $\angle CAB$.

Prove that $|\angle BAC| + |\angle CAD| + |\angle DAB| = 360°$. [*Hint:* Show that (CD) meets AB in a point E such that $E * A * B$ holds.]