

**UPDATED GENERAL INFORMATION — FEBRUARY 26, 2009**

**Information about the second examination.** As noted in class, this will cover Sections II.3 through III.3, but as usual in mathematics courses background from earlier parts of the course is likely to be covered in some form. The files of exercises and solutions are considered to be part of the material covered. There will be six problems with point values ranging from 10 to 20 points. Here is a rough breakdown into the types of questions; some problems on the exam may have parts of different types.

- (1) **Statements of definitions, assumptions or results.** Approximately one sixth of the exam.
- (2) **Giving reasons for certain conclusions, but not necessarily full formal proofs.** Approximately one third of the exam.
- (3) **Proving statements at the level of fairly short proofs in the notes or relatively uncomplicated homework exercises.** One fifth of the exam.
- (4) **Working out numerical examples which illustrate basic definitions, assumptions or results.** Approximately two fifths of the exam.

Here are more specific suggestions, including some problems which were considered but either not included or may appear in a simplified form.

- (1) Determine which points on a specific line lie in the interior of a specific angle. For example, let  $A = (0, 1)$ ,  $B = (-1, 0)$  and  $C = (0, -2)$ , and take the line to be  $x = 10$ . One can also ask similar questions about arbitrary points in the plane.
- (2) If a parallelogram's sides all have equal length, is it a square? Prove this or give a counterexample.
- (3) Prove that a parallelogram is a rectangle if and only if the lengths of its diagonals are equal.
- (4) Given  $\triangle ABC$ , show that  $\angle ACB$  is obtuse if and only if  $d(A, C)^2 > d(A, B)^2 + d(B, C)^2$ . Here is a simpler variant: Suppose that we are given  $\triangle ABC$  such that  $d(A, B) = 4$ ,  $d(A, C) = 7$  and  $d(B, C) = 5$ . If we are given that one of the angles in the triangle is obtuse, which one is it? This one requires no trigonometry.
- (5) Suppose that we are given  $A * B * C$  and  $D * E * F$ . Prove that if  $d(A, C) = d(D, F)$  and  $d(A, B) = d(D, E)$ , then  $d(B, C) = d(E, F)$ .
- (6) Suppose we are given  $\triangle ABC$  and  $D \in (BC)$  is such that  $[AD]$  bisects  $\angle BAC$ . If  $|\angle ABC| = x$  and  $|\angle ACB| = y$ , find  $|\angle ADC|$ .
- (7) If the plane  $P$  is defined by the equation  $2x + 3y + 4z = 9$  and  $X$  is the point  $(2, 1, 2)$ , find an equation defining the plane through  $X$  which is parallel to  $P$ .
- (8) Two sides of a triangle have lengths 12 and 2, while the third is an integral multiple of 3. Show that the triangle must be an isosceles triangle. [*Hint:* Use the Triangle Inequality to show there is only one possibility for the length of the third side.]
- (9) Show that in 3-dimensional space it is possible to have lines  $L, M, N$  such that  $L \cap M = \emptyset = M \cap N$  but  $L \cap N \neq \emptyset$ .