Math 133
Winter 2009

UPDATED GENERAL INFORMATION — MARCH 9, 2009

Misprint alert. In an earlier version of the file `geometrynotes5b.pdf`, on line 11 of page 261 (the next to the last line in the solution for Problem 1), the inequality should read "$|\angle DBC| < |\angle BCD|$" instead of "$|\angle DBC| < |\angle CBD|$." The online version of the file has been corrected accordingly .

Information about the third examination. As noted previously, this will cover Sections III.4–III.5 and V.2–V.4, but as usual in mathematics courses background from earlier parts of the course is likely to be covered in some form. The files of exercises and solutions are considered to be part of the material covered. There will be six problems with point values ranging from 10 to 20 points. Here is a rough breakdown into the types of questions; some problems on the exam may have parts of different types.

(1) **Statements of definitions, assumptions or results.** Approximately one fourth of the exam.

(2) **Proving statements at the level of fairly short proofs in the notes or relatively uncomplicated homework exercises.** Approximately two fifths of the exam.

(3) **Working out numerical examples which illustrate basic definitions, assumptions or results.** Approximately one third of the exam.

At the beginning of each question there will be a statement whether the underlying geometry is assumed to be **Euclidean**, **hyperbolic** or **neutral**.

Here are more specific suggestions, including some problems which were considered but either not included or may appear in a simplified form.

(1) Assume that we are working in a given **Euclidean** plane. — Suppose that ΔABC is isosceles with $d(A, B) = d(A, C)$, and let D be the midpoint of $[BC]$. Explain why each of the centroid, circumcenter, orthocenter and incenter lie on the line AD. Describe examples for which all four of these points lie on $[AB]$, and also describe examples for which at least one of these points does not lie on the open segment (AB).

(2) Assume that we are working in a given **Euclidean** plane. — Suppose that we are given ΔABC, and let $D \in (AB)$ be such that $\Delta ADC \sim \Delta CDB$. Prove that ΔACB is a right triangle, and furthermore we have $\Delta ADC \sim \Delta ACB$ and $\Delta CDB \sim \Delta ACB$.

(3) Assume that we are working in a given **Euclidean** plane. — Suppose that we are given ΔABC and ΔDEF, and also assume that there are points $G \in (BC)$, $H \in (EF)$ such that $\Delta ABG \sim \Delta DEH$ and $\Delta AGC \sim \Delta DHF$. Prove that $\Delta ABC \sim \Delta DEF$.

(4) Assume that we are working in a given **Euclidean** plane. — Suppose that we are given lines L and M with distinct points A, B, C, D, E, F such that the first three points are on L and satisfy $A * B * C$, while the second three points are on M and satisfy $D * E * F$. Furthermore, assume that the three lines AD, BE and CF are all parallel to each other. Prove that

$$\frac{d(A,B)}{d(B,C)} = \frac{d(D,E)}{d(E,F)} .$$

[*Hint:* Use vectors and write $C = A + t(B - A)$ and $F = D + u(E - D)$ for suitable scalars. What do the betweenness relations say about t and u separately? How do the parallelism conditions imply an equation involving t and u?]

Also consider the following converse problem: If two of the three lines are parallel and the proportionality equation is valid, prove that the third line is parallel to the other two.

(5) Assume that we are working in a given **neutral** plane. — Suppose that ΔABC is isosceles with $d(A, B) = d(A, C)$, let E and F be the midpoints of $[AB]$ and $[AC]$ respectively, and suppose that D is the midpoint of $[BC]$. Prove that the lines AD and EF are perpendicular. [*Note:* At some point in the proof it will probably be necessary to show that AD and EF have a point in common.]

(6) Assume that we are working in a given **neutral** plane. — Suppose that ΔABC is given and that $B * C * D$ is true. Prove that $|\angle BCD| \geq |\angle BAC| + |\angle ABC|$. Can we state a stronger conclusion if the plane is hyperbolic? Give reasons for your answer.

(7) Assume that we are working in a given **hyperbolic** plane. — Using the exercises for Section V.3 and results from Section V.4, explain why a Saccheri quadrilateral is never a Lambert quadrilateral and vice versa. [*Hint:* In neutral geometry, explain how the exercises imply that a convex quadrilateral which is both a Saccheri quadrilateral and a Lambert quadrilateral must be a rectangle.]