

ANOTHER ARC LENGTH EXAMPLE

$$\gamma(t) = (t + \sin t, \cos t) \quad 0 \leq t \leq \pi$$

$$\gamma'(t) = (1 + \cos t, -\sin t)$$

$$|\gamma'(t)| = \sqrt{(1 + \cos t)^2 + \sin^2 t} = \sqrt{2 + 2\cos t}$$

Simplify using trig identity

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} \quad \text{for } 0 \leq \theta \leq \pi \Rightarrow$$

$$|\gamma'(t)| = 2 \cos \frac{t}{2}$$

Find the length of γ from time 0 to time t :

$$s(t) = \int_0^t 2 \cos \frac{u}{2} du = 4 \int_0^t \cos \frac{u}{2} d\frac{u}{2} \quad \underline{\underline{v = \frac{u}{2}}}$$

$$4 \int_0^{t/2} \cos v dv = 4 \sin t/2.$$

Now find the arclength parametrization s :

Solve for t in terms of s .

$$s = 4 \sin t/2 \Rightarrow s/4 = \sin t/2 \Rightarrow \text{Arcsin } s/4 = t/2 \Rightarrow$$

$$t = 2 \text{Arcsin}(s/4).$$