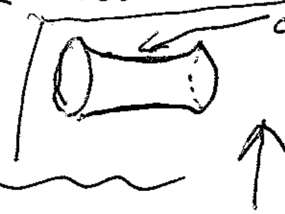
Catenoid

Recall: $\cosh x = \frac{1}{2}(e^{x}+e^{-x})$

$\sinh x = \frac{1}{2}(e^{x}-e^{-x})$

so that $D\cosh x = \sinh x$, $D\sinh x = \cosh x$ and

$\cosh^2 x - \sinh^2 x = 1.$



Catenoid = solid of revolution about x-axis corresponding to the curve $y = \cosh x$.

So $X(u,v) = (u, \cosh u \cos v, \cosh u \sin v)$.

$X_1 = (1, \sinh u \cos v, \sinh u \sin v)$

$X_2 = (0, -\cosh u \sin v, \cosh u \cos v)$

$X_1 \times X_2 = \Omega = (\sinh u \cosh u, -\cosh u \cos v, -\cosh u \sin v)$

$|\Omega|^2 = \sinh^2 u \cosh^2 u + \cosh^2 u = (\sinh^2 u + 1)\cosh^2 u = \cosh^4 u.$

So $N = \frac{1}{\cosh^2 u} \cdot \Omega$ (this can be reduced further, but the given form is easier to manipulate!)

$$H = \frac{eG - 2fF + gE}{2(EG - F^2)}.$$ (notes, p. 95)

Substitute into this expression

$$\frac{-\cosh^3 u - 0 + \cosh^3 u}{2\cosh^4 u} = 0$$

So this catenoid is a minimal surface.

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See Theorem 7.2 in O'Neill for an important theorem involving catenoids.

CAT 2

FFF

$$E = 1 + \sinh^2 u = \cosh^2 u.$$

$$F = 0$$

$$G = \cosh^2 u \qquad EG - F^2 = \cosh^4 u.$$

SFF

$$X_{11} = (0, \cosh u \cos v, \cosh u \sin v)$$

$$X_{12} = (0, -\cosh u \sin v, \cosh u \cos v)$$

$$X_{22} = (0, -\cosh u \cos v, -\cosh u \sin v)$$

recall

$$N = \frac{1}{\cosh u} (\sinh u, -\cos v, -\sin v)$$

Hence $e = \frac{1}{\cosh u} - \cosh^2 u = -\cosh u$

$$g = \cosh u$$

$$f = 0$$

So $K = \frac{-\cosh^2 u}{\cosh^4 u} = -\frac{1}{\cosh^2 u}.$