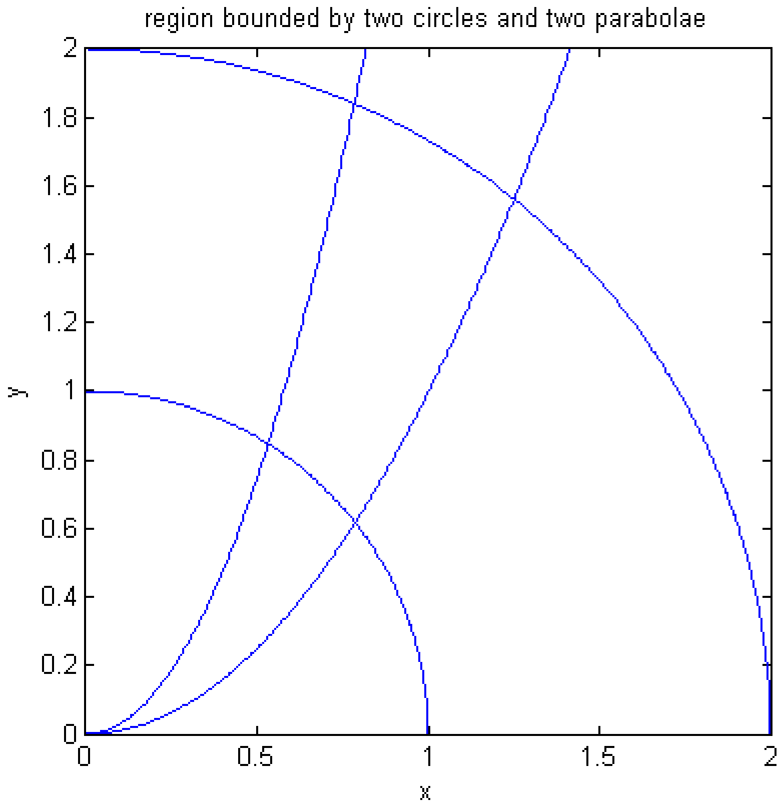
Another change of variables transformation

http://www2.math.umd.edu/~jmr/241/changevar.html

The change of variables formula for multiple integrals is an extremely effective way of simplifying the computation of such objects. Frequently the crucial problem is to find a transformation T which maps some very simple region – for example, a solid rectangle – into a given region over which we want to integrate some function. In the drawing below, the given region is bounded by the circles with equations $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, and the parabolas with equations $y = x^2$ and $y = 3x^2$.



If we make the change of variables $u = x^2 + y^2$ and $v = y/x^2$, then the inverse transformation $(x, y) = T(u, v)$ to $(u, v) = (x^2 + y^2, y/x^2)$ will send the solid rectangular region bounded by the four lines

$$u = 1, \quad u = 4, \quad v = 1, \quad v = 3$$

to the displayed region bounded by the circular and parabolic arcs. We can find T explicitly by solving the change of variables equations for x and y in terms of u and v:

$$y = \frac{1 + \sqrt{1 + 4v^2}}{2v}, \qquad x = \sqrt{\frac{1 + \sqrt{1 + 4v^2}}{2v^2}}$$