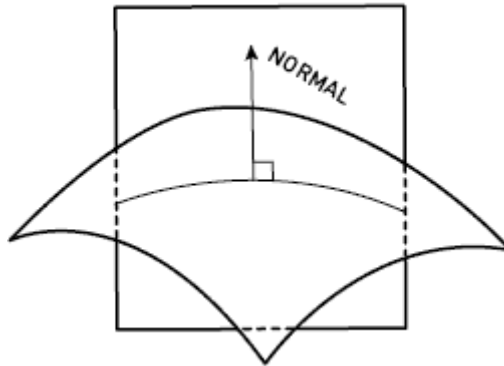


Plane sections of a surface

It is sometimes useful to analyze surfaces by means of *plane sections*, which are given by intersecting the surface with a plane which contains a normal line to a point \mathbf{p} on the surface. The drawing below suggests that such an intersection is a curve, and in fact this is always true, at least near the point where the surface and line meet each other.



(Source: <http://www.learner.org/courses/mathilluminated/images/units/8/1832.png>)

We can use the results of Section III.6 to prove the intersection is locally a curve because the surface and the plane intersect transversely. The latter is true because the plane contains the normal line to the surface at the point \mathbf{p} , which means that the normal to the plane is perpendicular to the normal to the surface at \mathbf{p} .