Winter 2012

# Review Suggestions for the Math 138A Final Examination

About 25% of the exam will cover the material for the first exam as summarized in review1.pdf
($\Rightarrow$ will not be repeated here)

1. Know the statements of the inverse and implicit function theorems and how to apply them (e.g., show that one can solve $(a, b) = (u^2 - v^2, 2uv)$ for all points close to $(r, 0)$, where $r > 0$). Know how to use the Chain Rule to compute derivative matrices of composite functions.

   Examples

   $$(x, y) = F(u, v) = (u + v^2, v)$$
   $$G(x, y) = (x^2 - y^2, 2xy)$$
   $$\text{or } \frac{1}{x^2+y^2}(x, y).$$

   In particular, understand the use of the Jacobian

25% of exam on Unit I, 25% on II
50% on III & IV

2. Know the two basic approaches to describing surfaces — regular parametrization and level-sets — and the conditions needed to ensure that $X = \sigma(u,v)$ or $F(x,y,z) = 0$ is "good" for defining a surface. Know how to define tangent planes in each approach and understand the concept of the 2D vector subspace of tangent vectors at a point $p$ in a surface $\Sigma$ (from both approaches)

It is not necessary to know explicitly how these can be related using the concept of a regular geometric surface, but it is good to know ~~that~~ how the tangent plane/vector concepts are related in the two approaches. As is often the case, planes and 2D subspaces of interest can be characterized ~~using~~ by knowing a point on such a set and the normal direction (how?).

$z = u, v$

~~and~~ $E, F, G$ in terms of $\frac{\partial \sigma}{\partial z}$ ).

3. Know how to compute the First Fundamental Form. Know what it means to have a (normal) orientation for a surface and how these can be constructed from the $X = \sigma(u,v)$ and $F(x,y,z)=0$ descriptions of surfaces. Know the definition of Gauss maps and be able to compute well enough to see that sometimes the Gauss map is constant, sometimes it is 1-1 onto, and sometimes it is nonconstant but not onto (for the latter, look at examples like the surfaces $z = x^2 \pm y^2$).

4. Know the definition of the Shape Operator in a parametrization $\left(\frac{\partial\sigma}{\partial z} \to \frac{\partial N}{\partial z}\right)$ for an oriented surface, know that it is self adjoint at each point $p$ in the surface $\Sigma$ and what this means in terms of eigenvectors and eigenvalues. Know the definition of the Second Fundamental Form in terms of the Shape Operator, know the "easy" formulas for computing $e, f, g$ in terms of $N$ and the second partials of $\sigma$. Know how to compute the Gaussian curvature using FFF and SFF. Be able to carry out these computations for simple examples of quadric surfaces if the parametrization is given or easy to see (e.g., if $z = xy$, take $\sigma(u,v) = (u, v, uv)$). In cases where the Shape Operator takes a simple form, be able to find the Gaussian curvature and mean curvature directly (e.g., for the sphere or a plane, the shape operator is a scalar mult. of the identity, so the eigenvalues can be read off without much additional work).