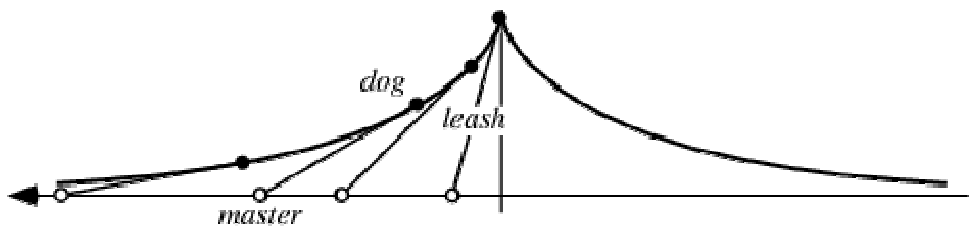
The Tractrix Curve

The opening statement of the ***Wikipedia*** article (**http://en.wikipedia.org/wiki/Tractrix**) summarizes things very well**:**

> [The] ***tractrix*** (from the Latin verb *trahere* "pull, drag"**;** plural**:** ***tractrices***) is the curve along which an object moves**,** under the influence of friction**,** when pulled on a horizontal plane by a line segment attached to a tractor (pulling) point that moves at a right angle to the initial line between the object and the puller at an infinitesimal speed**.** It is therefore a curve of pursuit**.** It was first introduced by Claude Perrault in 1670**,** and later studied by Sir Isaac Newton (1676) and Christian Huygens (1692)**.**



(*Source:* http://mathworld.wolfram.com/Tractrix.html)

There is an animated version of this drawing in the ***Wikipedia*** article**.** The following parametric equations are given in **http://xahlee.org/SpecialPlaneCurves_dir/Tractrix_dir/tractrix.html:**

(Log(Sec[t] + Tan[t]) - Sin[t]**,** Cos[t]**),** $-\pi/2 < t < \pi/2$.

As indicated in Unit **IV** of the course notes**,** the surface of revolution obtained by rotating the tractrix around the x – axis**,** which is called the ***pseudosphere***, has **constant negative Gaussian curvature.**

FOOTNOTE. Since the Riemannian metric for the classical non – Euclidean (or ***hyperbolic***) plane also has constant negative Gaussian curvature**,** it is useful to explain the relationship between hyperbolic and pseudospherical geometry**.** Although the geometry of the pseudosphere is not the same as classical non – Euclidean (or ***hyperbolic***) geometry**,** suitably chosen small pieces of each turn out to have the same intrinsic geometrical properties (in the same sense that small pieces of a plane and cylinder have the same intrinsic geometrical properties)**.**