Mathematics 138A, Winter 2010, Take Home Assignment

This will be due at the beginning of class on
Friday, March 5, 2010.

You must show the work behind or reasons for your answers. The point values are indicated in parentheses.

1. (*20 points*) Let U be an open set in $\mathbb{R}^3$, and let $P(x,y,z)$ and $Q(x,y,z)$ be function with continuous partial derivatives on U, and let ω and θ be the 1-forms $P(x,y,z)\,dx$ and $Q(x,y,z)\,dy$ respectively. Prove that exterior differentiation satisfies the identity

$$d(\omega \wedge \theta) \quad = \quad d(\omega) \wedge \theta \; - \; \omega \wedge d(\theta) \; .$$

The negative sign is **not** a misprint.

2. (*20 points*) Let $f(t)$ be a smooth function. Show that the tangent planes to the surface $z = xf(y/x)$, where $x \neq 0$, all pass through the origin. [*Hint:* A plane passes through the origin if and only if it is definable by an equation of the form $ax + by + cz = 0$, where at least one of a, b, c is nonzero.]

3. (*20 points*) Compute the First Fundamental Forms of the following parametrized surfaces:

(a) The the cone $(z\cos v, z\sin v, z)$, where $z \neq 0$.

(b) The corkscrew surface $\mathbf{X}(r,\theta) = (r\,\cos\theta,\, r\,\sin\theta, \theta)$, where $r > 0$.

4. (*20 points*) Let $(x,y,z) = \mathbf{f}(u,v,w) = (u^2 - uv, v - u, w^2 - wv)$ be a vector valued function of 3 variables. Then one of the following two statements is true:

(a) For all (x,y,z) sufficiently close to $(0,0,0)$, the system of equations $(x,y,z) = \mathbf{f}(u,v,w)$ has a unique solution close to $(1,1,1)$.

(b) For all (x,y,z) sufficiently close to $(2,-2,2)$, the system of equations $(x,y,z) = \mathbf{f}(u,v,w)$ has a unique solution close to $(1,-1,1)$.

Using the Inverse Function Theorem, determine which of these statements is true.