Mathematics 138A

Introduction to Differential Geometry — I

Course Notes

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Department of Mathematics

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