Boundaries and partial differentiation

This is yet another attempt to indicate why one sticks with open regions when discussing partial differentiation, in contrast to ordinary differentiation where one can work equally well with open, closed and half – open intervals.

The short answer is that there are many more possibilities for the shapes of boundaries for regions in two or more dimensions, and usually this complicates matters considerably. Some examples are given in the following document:

http://math.ucr.edu/~res/math138A/opensets.pdf

A few of the possibilities for boundaries are obvious, like the nice flat boundary for the upper half plane (nonnegative second coordinate) or the first quadrant (both coordinates nonnegative). However, boundaries can be considerably more complicated than either of these. For example consider the Koch snowflake fractal curve in the plane and the region it bounds. Here is an animated description of this curve:

http://en.wikipedia.org/wiki/File:Von_Koch_curve.gif

There are many other examples of bizarre boundaries, and it turns out that the easiest way to address the issue of partial differentiation is to formulate everything so that boundary points are avoided.