Math 138A–010
Winter 2010

UPDATED GENERAL INFORMATION — FEBRUARY 1, 2010

Here are some comments regarding the midterm examination which was rescheduled for **Monday, February 8**.

The exam will cover Sections I.1 – I.5 and II.2 – II.4 of the online course notes, the exercises for these sections in the file `dgexercises2010x2.pdf` and the corresponding material in O'Neill as indicated in the notes. This coverage will include the material in the following course directory files:

```
frenetnote.pdf
zeropartials.pdf
opensets.pdf
neighborhoods.pdf
cyc-curves.pdf
skewlines10.pdf
linapprox.pdf
changevarexamples.pdf
curvexamples.pdf
problem125.pdf
helix.pdf
helix2.pdf
congruence000.pdf
colleyp170.pdf
```

The following deal with supplementary topics not covered on the exam, but they provide some details that play significant roles but are not discussed explicitly in the course:

```
expmatrix.pdf
rigidmotions.pdf
```

The exam itself will consist of five problems. Some may be close or identical to assigned problems from the text or unstarred problems in the supplementary exercises.

More precisely, this exam will contain problems on some of the following topics: Arc length and related reparametrizations of curves, definitions of principal normals, binormals, curvature and torsion, computing curvature and torsion, knowledge of which curves have constant curvature and torsion (this splits into cases depending upon whether such constants are zero or nonzero separately), the statement of the Fundamental Theorem on Curves, definition of an open set in coordinate n-space, the derivative matrix of a vector valued function of n variables and its linear approximation property, the Chain Rule, the statements of the Implicit and Inverse Function Theorems, the use of these results in working with specific functions, and the notions of isometry and congruence in coordinate n-space defined using concepts from linear algebra.

In addition to exercises in the documents listed above and mentioned in the lectures, the previously cited file `colleyp170.pdf` has some worthwhile practice problems involving the material from Section II.3 of the notes, and exercises 8 - 11 on pages 4–5 of the file

`http://math.ucr.edu/~res/math133/math133exercises2.pdf` are strongly recommended in connection with Section II.4.

Here are a couple of problems that were considered for the exam; the ability to work them successfully is a good measure of how well the course material through Unit II is understood.

> Suppose that $\gamma(s)$ is a smooth curve with a modified arclength parametrization, so that $|\gamma'| = 1$ always, and let $a > 0$. Let $\beta(s)$ be the dilated curve $a \cdot \gamma(s)$. — Show that $\beta(s/a)$ defines a modified arc length parametrization of β, and using this give a formula for the curvature of β at $s = s_0$ in terms of the curvature of γ at $s = s_0$. [*Hint:* What happens for the circle?]. Also do the same thing for the torsions of the respective curves.
>
> Consider the transformation
>
> $$\mathbf{F}(x,y,z) \;=\; \left(\frac{1}{x} + \frac{1}{y+z}, \frac{1}{y} + \frac{1}{x+z}, \frac{1}{z} + \frac{1}{x+y}\right)$$
>
> for which one has $\mathbf{F}(1,1,1) \;=\; (\frac{3}{2}, \frac{3}{2}, \frac{3}{2})$. Show that for each (u,v,w) sufficiently close to $(\frac{3}{2}, \frac{3}{2}, \frac{3}{2})$ there is a solution to the vector equation $\mathbf{F}(x,y,z) \;=\; (u,v,w)$.

GENERAL REMARKS. The ability to sketch simple curves and regions may be helpful for analyzing problems and finding answers.

Unless indicated otherwise, the logical steps in solving problems should be shown to ensure the maximum possible credit; partial credit will be given for incorrect answers in some cases, depending upon the extent to which the work shown on the exam is valid.

No electronic computing devices will be necessary, and none will be permitted. Likewise, no open books or notes will be permitted.