Math 138A–010
Winter 2010

UPDATED GENERAL INFORMATION — MARCH 9, 2010

Here are some comments regarding the fina examination which is scheduled for **Tuesday, March 16**.

The exam will cover Units I – IV of the online course notes (omitting Section IV.6), the exercises for these sections in the file `dgexercises2010x2.pdf`, the supporting files in the course directory and the corresponding material in O'Neill as indicated in the notes.

About a third of the exam will cover earlier topics from the first two units, including curvature and torsion, the computation of these for relatively simple examples, the derivative matrix of a vector valued function of several variables, its linear approximation properties, and results involving the derivative matrix and jacobian like the Inverse and Inverse Function Theorems, and the basic properties of differential forms.

The remainder of the examination will cover material on surfaces from Units III and IV. The first part includes the general concept of surface, presentations of surfaces (or parts of surfaces) in terms of regular parametrizations or sets of the form $f(x, y, z) = 0$ where f satsifies a basic regularity condition (if $f(a, b, c) = 0$, then its gradient at (a, b, c) is nonzero), and background concepts mainly from multivariable calculus including tangent planes, normal directions at points of a smooth surface, and the 2-dimensional vector space of tangent vectors at a point in the surface. The next level includes the concept of First Fundamental Form, including the description of the latter in terms of a parametrization, and the results on curves formed by transversely intersecting surfaces; in particular, if you are given a point $\mathbf{p}$ which lies on two specific transversely intersecting surfaces, you should know how to find the direction of the tangent vector to that curve from the surface data. The next level involves the idea of an orientation vector field (a continuous choice of unit normal vectors), the formulas for the latter in terms of local coordinates, the definition of the Shape Operator and Second Fundamental Form, computation of the latter in terms of a parametrization (including useful formulas for doing so), and formulas for computing the mean and Gaussian curvatures of surfaces in terms of the First and Second Fundamental forms (the Gaussian curvature is particularly important).

In particular, you should be able to work out all the basic differential geometric objects associated to a surface with a given regular parametrization, including the unit normal vector fields (locally there are two of them), the First and Second Fundamental Forms, and the Gaussian curvature. There will be no single problem on the exam asking for everything, but exercises asking for partial computations are certain to be on the examination. Working with such examples in one way or another will be the main thing asked for in the problems in surfaces.

Finally, you will be expected to know something about the concepts and results involving matrices that were discussed in connection with self-adjointness for matrices and linear transformations, including the existence of an orthonormal basis of eigenvectors (sent to scalar multiples of themselves), and the traces and determinants of matrices and linear transformations (at least in the 2×2 case.

Suggested exercises. The exercises in the `dgexercises2010` documents are obvious suggestions, but some of them are difficult. Looking through some of the handwritten computations and supplementary documents on computing examples (curvature and torsion for specific curves,

Fundamental Forms and Gaussian curvature of surfaces) is very strongly recommended. Also, the problems in the files `math138Aexam*.pdf`, where $*$ is one of 1, 2 or 3A, would be worthwhile.