

Wedge products and exterior derivatives

Here is another exercise regarding wedge products, this time concerning identities for wedge products and exterior derivative.

EXERCISE. (a) Suppose that ω is either a 1-form or a 2-form and f is a smooth function. Prove that $d(f\omega) = df \wedge \omega + f d\omega$.

(b) Suppose that ω and θ are both 1-forms. Prove that $d(\omega \wedge \theta) = (d\omega) \wedge \theta - \omega \wedge d\theta$. (Note the minus sign.)

These formulas can be verified using the standard expansions of 1-forms and 2-forms like $P dx + Q dy + R dz$, the basic identities such as $d(P dx) = dP \wedge dx$ and $df = f_x dx + f_y dy + f_z dz$, and the rearrangement identities of the form $du \wedge du = 0$, $dv \wedge du = -du \wedge dv$.