

**Mathematics 138A**

**Introduction to Differential Geometry — I**

**Course Notes**

**Winter 2004**

**Department of Mathematics**

**University of California, Riverside**

## Table of Contents

|      |   |     |
|------|---|-----|
| I.   | <b>Classical differential geometry of curves</b>  | 1   |
| 1.   | Cross products                                    | 3   |
| 2.   | Parametrized curves                               | 7   |
| 3.   | Arc length and reparametrization                  | 12  |
| 4.   | Curvature and torsion                             | 15  |
| 5.   | Frenet-Serret Formulas                            | 27  |
| II.  | <b>Closed curves as boundaries</b>                | 35  |
| 1.   | Regions, limits and continuity                    | 37  |
| 2.   | Smooth mappings                                   | 40  |
| 3.   | Inverse and implicit function theorems            | 45  |
| 4.   | Global properties of plane curves                 | 48  |
| III. | <b>Surfaces in 3-dimensional space</b>            | 55  |
| 1.   | Mathematica descriptions of surfaces              | 55  |
| 2.   | Parametrizations of surfaces                      | 57  |
| 3.   | Tangent planes                                    | 66  |
| 4.   | The First Fundamental Form                        | 70  |
| 5.   | Surface area                                      | 74  |
| 6.   | Curves as surface intersections                   | 76  |
| IV.  | <b>Oriented surfaces</b>                          | 79  |
| 1.   | Normal directions and Gauss maps                  | 79  |
| 2.   | The Second Fundamental Form                       | 82  |
| 3.   | Quadratic forms and adjoint transformations       | 86  |
| 4.   | Normal, Gaussian and mean curvature               | 89  |
| 5.   | Special classes of surfaces                       | 94  |
| 6.   | Compatibility equations, <i>Theorema Egregium</i> | 100 |
| 7.   | Fundamental Theorem of Local surface theory       | 107 |