Math 144
Fall 2019

UPDATED GENERAL INFORMATION — NOVEMBER 1, 2019

More practice problems for the midterm examination

Solutions will be posted later.

1. If A and B are subsets of some larger set X, then the *symmetric difference* $A+B$ is given by $(A-B)\cup(B-A)$ (see also the first exercise in `aabUpdate03.144.f17.pdf`).

(a) Prove that the symmetric difference satisfies the identities $A+\emptyset = A$ and $A+A=\emptyset$ for all subsets A.

(b) Prove that the symmetric difference satisfies an associative law: $A+(B+C) = (A+B)+C$. One way of attacking this problem is to consider the eight mutually exclusive cases depending upon whether or not x belongs to A, B or C and note that for each of the eight cases the element x either belongs to both of the sets in question or to neither of them.

(c) Prove that if $A+C = B+C$ then $A = B$. [*Hint:* Add C to both sides.]

2. Let A be a nonempty set. Prove that

$$\bigcup_{a\in A}\{a\} \quad = \quad A\ .$$

What can we say about $\cap_{a\in A}\ \{a\}$? [*Hint:* One point sets of the form $A=\{a_0\}$ are an exceptional case.]

3. Let $A = B\times B$ where B is a finite set with at least two elements, and define a binary relation $\mathcal{W}$ on pairs by $(x,y)\sim(u,v)$ if and only if $\{x,y\}\cap\{u,v\}$ is nonempty (informally, we are looking at words with two letters in the alphabet B and saying that two words are related if they have a letter in common).

(a) Is $\mathcal{W}$ an equivalence relation? Verify this or show why it is not true.

(b) Let $\mathcal{E}$ be an equivalence relation such that two words which are $\mathcal{W}$-related are $\mathcal{E}$-equivalent. Are all words $\mathcal{E}$-equivalent to each other? Prove this or give and example where the answer is negative.

4. In this exercise all subsets of the real numbers are assumed to be equipped with the linear ordering they inherit from $\mathbb{R}$: For each of the intervals

$$J \quad = \quad [0,1]\ , \quad [0,1)\ , \quad (0,1]\ , \quad (0,1)$$

either prove that J is order-isomorphic to $[1,2]$ or give a reason why J cannot be order-isomorphic to $[1,2]$.

5. For each of the functions

$$f(x) \quad = \quad \frac{1}{x^2+1}\ , \qquad \frac{1}{2+\sin x}\ , \qquad \frac{1}{e^x+3}$$

(i) find an interval $A=[a,b]$ so that $f|A$ is 1–1, (ii) find an interval $B=[p,q]$ such that B is contained in $f[R]$.

6. Let $f(x) = x^n$ where x is a real number. Find the inverse image $f^{-1}[A]$ where A is the interval $[0,1]$. [*Hint:* There are two cases depending upon whether n is even or odd.]