Math 144
Fall 2019

UPDATED GENERAL INFORMATION — DECEMBER 4, 2019

More study exercises for the second examination

0. Given sets A and B, prove the cardinal number inequality $|A \cap B| \leq |A|$.

1. (a) Let $f : A \to B$ and $g : B \to C$ be functions such that $g \circ f$ is 1–1 and onto. Prove that f is 1–1 and g is onto.

(b) Let $\mathcal{V}$ and $\mathcal{W}$ be equivalence relations on a set S, and define a new relation $\mathcal{Y}$ on S such that $a \; \mathcal{Y} \; b$ if and only if $a \; \mathcal{V} \; b$ and $a \; \mathcal{W} \; b$. Prove that $\mathcal{Y}$ defines an equivalence relation on S.

2. Let $\mathbb{N}_-$ be the set of all **negative** integers with the usual linear ordering. Is $\mathbb{N}_-$ well-ordered? Either prove this is true or explain why it is false.

3. Determine the cardinality of the set of all open intervals $(a, b) \subset \mathbb{R}$, where $a, b \in \mathbb{R} \cup \{-\infty, +\infty\}$.

4. Prove the following formula for all $n \geq 2$ by mathematical induction:

$$\sum_{k=2}^{n} \frac{1}{k^2 - k} \quad = \quad 1 \;-\; \frac{1}{n}$$

5. Let $f : X \to Y$ be a function, and let $C + D$ denote the symmetric difference of the subsets $C, D \subset Y$. Show that $f^{-1}[C + D] = f^{-1}[C] + f^{-1}[D]$.