Math 144
Fall 2017

UPDATED GENERAL INFORMATION — OCTOBER 20, 2017

The first quiz

The first quiz will take place in the discussion sections on Tuesday, October 24. It will consist of one question, consist of one problem at the level of recommended homework assignments, and only use up part of the period(s). Different discussion sections will probably have different questions, but the intent is for them all to be at the same level. A few practice problems are given below; the longer ones ask more than will be expected on the quiz, but it is always good to be prepared for something that is a little more challenging than one expects.

1. Given two sets X and Y, their *symmetric difference* $X + Y$ is the set $(X - Y) \cup (Y - X)$. There is a reason for adopting the plus sign, but in this problem the important thing to keep in mind is that it is not the same as the union of the two sets.

(a) Let $A = \{a, b, f, g\}$, let $B = \{b, c, d, g\}$, and let $C = \{d, e, f, g\}$. Verify that $(A+B)+C = A + (B + C)$ using the definitions.

(b) In the setting above, also verify that $(A + B) \cap C = (A \cap C) + (B \cap C)$.

2. (a) Suppose $X = \{a, b\}$ and $Y = \{x, y\}$. List the elements of the set $\mathbf{P}(X \times Y)$ which is the collection of all subsets $A \subset X \times Y$.

(b) Let $A = \{1, 2\}$ and $B = \{2, 3\}$. Find each $\mathbf{P}(A)$, $\mathbf{P}(B)$ and $\mathbf{P}(A \cup B)$. Explain why the first two are contained in the third; it follows that the union of the first two is also contained in the third. Is this containment proper? Prove it or give a counterexample.

3. (a) List all of the subsets of $\{1, 2, 3, 4, 5, 6\}$ which contain exactly two elements.

(b) List all of the subsets of $\{1, 2, 3, 4, 5, 6\}$ which contain exactly three elements.

4. Let $C = \{n \in \mathbf{Z} \mid n = 6r - 5 \text{ for some integer } r\}$ and $D = \{m \in \mathbf{Z} \mid m = 3s + 1 \text{ for some integer } s\}$. Determine whether $C \subset D$ or $D \subset C$, giving reasons for your answer.

4. Let A, B, C be subsets of some large set X. Is $A \cap (B - C) = (A \cap B) - (A \cap C)$? Prove it or give a counterexample.

5. (a) Let A, B, C be subsets of some large set X. Prove that if $A \subset B$ then $(A \cup C) \subset (B \cup C)$.

(b) Let A, B, C be subsets of some large set X. Prove that if $A \subset B$ then $(A \cap C) \subset (B \cap C)$.

6. Let E be a set, and let $A, B \subset E$. Prove that $(E \times E) - (A \times B) = (\,(E - A) \times E) \cup (E \times (E - B)\,)$. One way to start analyzing this problem is to draw a picture where E is equal to the real line and A, B are subintervals.

7. Let A, B, C be subsets of some large set X. Prove that X is the union of the sets

$$A \cap B \cap C, \quad A \cap B \cap (X - C), \quad A \cap (X - B) \cap C, \quad A \cap (X - B) \cap (X - C),$$

$$(X - A) \cap B \cap C, \quad (X - A) \cap B \cap (X - C), \quad (X - A) \cap (X - B) \cap C, \quad (X - A) \cap (X - B) \cap (X - C)$$

and this family of sets is pairwise disjoint.