Math 144
Fall 2017

UPDATED GENERAL INFORMATION — OCTOBER 25, 2017

The first in-class examination

The first in-class examination will take place in the lecture sections on Wednesday, November 1. It will consist of four problems. A few practice problems are given below; they cover more material than can be included in the examination.

As noted in the lectures, the cutoff point for material covered on the exam is the end of Section IV.1, and the starting point is Section II.2.

1. Given a set X, let $\mathbf{P}(S)$ denote the set of all its subsets. Prove that ir A is a subset of X then there is a subset $B \subset X$ such that $\mathbf{P}(X) = \mathbf{P}(A) \cup \mathbf{P}(B)$. [*Question:* Can we take $B = X - A$?]

2. Suppose that A, B, C are subsets of the set X. Prove that

$$(X - A) \ \cap \ (X - (B \cup C))) \quad = \quad (A \cup (X - B)) \cap (A \cup (X - C)) \ .$$

[*Hint:* For problems like this the substitution $Y^c = X - Y$ makes the algebraic manipulations much easier to manage.]

3. (a) Determine if the binary relation on real numbers given by $xNy \quad \Leftrightarrow \quad |y - x| < 1$ is (i) symmetric, (ii) reflexive, (iii) transitive.

(b) Let A be an indexing set, and let Y_α be a family of subsets indexed by the elements of A. Prove that

$$\bigcup_{\alpha \in A} X \times Y_\alpha \quad = \quad X \times \left(\bigcup_{\alpha \in A} Y_\alpha \right) \ .$$

4. Suppose that X is a set, $A \subset X$ and $\mathbf{P}(X)$ is the set of all subsets of X. Which, if any, of the following is/are always true? Which, if any, are sometimes true? Which, if any, are never true?

(a) $A \in \mathbf{P}(X)$.

(b) $\{A\} \in \mathbf{P}(X)$.

5. Let R be an equivalence relation on $A = \{a, b, c, d, e, f, g, h\}$, and suppose that we have

$$aRb, \ cRb, \ cRd, \ eRd, \ eRf, \ gRf \ .$$

Prove that aRg.

6. Give an example of a finite set A and an equivalence relation R on A which has equivalence classes containing precisely 1, 2, 3 and 4 elements. [*Hint:* Find a partition with this property.]

7. Let R and S be equivalence relations on a set A, and let T be the relation $xTy \ \Leftrightarrow \ xRy$ and xSy. Prove that T is also an equivalence relation on A.

8. One of our axioms for set theory is that there is no set A such that $A \in A$. Using this, explain why the sets $\{\{\emptyset\}\}$ and $\{\{\{\emptyset\}\}\}$ are distinct.