Math 144
Fall 2017

UPDATED GENERAL INFORMATION — NOVEMBER 9, 2017

Returning the first in-class examination

The examination has been graded and the curve set. A second message will be posted later today or tomorrow at the latest; information for the second quiz has higher priority.

The second quiz

The second quiz will take place in the discussion sections on Tuesday, November 14. It will consist of one question, consist of one problem at the level of recommended homework assignments, and only use up part of the period(s). The coverage will be material from Sections IV.3 and IV.4; the only possible use of partial orderings would involve the usual linear ordering on the real numbers. Different discussion sections will probably have different questions, but the intent is for them all to be at the same level.

1. The definition of a one-to-one function $f : X \to Y$ is stated in two ways:

For all $x_1, x_2 \in$, if $f(x_1) = f(x_2)$ then $x_1 = x_2$.

For all $x_1, x_2 \in$, if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$

Why are these two statements logically equivalent?

2. Let $f : \mathbb{Z} \to \mathbb{Z}$ be the function $f(n) = 2n$.

(a) Is f 1–1? Is f onto? In each case, give a proof if the answer is yes and a counterexample if the answer is no.

(b) What happens to the preceding if we change the codomain to $2\mathbb{Z}$?

3. As usual let $|x|$ denote the absolute value of a real number x, and let $f : \mathbb{R} \to \mathbb{R}$ be the function $f(x) = x \cdot |x|$. Is f 1–1? Is f onto? In each case, give a proof if the answer is yes and a counterexample if the answer is no.

4. Find the mistake in the following proof that the function $f : \mathbb{R} \to \mathbb{R}$ defined by the formula $f(x) = 4x + 3$ is one-to-one: *Suppose any real number x is given. Then by definition of f, there is only one possible value for $f(x)$; namely, $4x + 3$. Hence f is one-to-one.*

5. In the examples below, a function f is defined on the set of real numbers such that the denominator is nonzero. In every case, determine whether or not f is one-to-one and justify your answer:

$$\frac{x+1}{x}\,, \qquad \frac{x}{x^2+1}\,, \qquad \frac{3x-1}{x}\,, \qquad \frac{x}{x^2+1}$$

6. The sum $A = f + g$ of two real valued functions is defined by $A(x) = f(x) + g(x)$.

(a) If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

(b) If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are both onto, is $f + g$ also onto? Justify your answer.

7. In the examples below, functions f and g from X to itself are defined by formulas. Find the composites $f \circ g$ and $g \circ f$ and determine if they are equal:

(a) $X = \mathbb{R}$, $f(x) = x^3$ and $g(x) = x - 1$.

(b) $X = \mathbb{R}$, $f(x) = x^5$ and $g(x) = x^{1/5}$.

(c) $X = \mathbb{Z}$, $f(n) = 2n$ and $g(x) = [n/2]$, where $[x]$ denotes the greatest integer $\leq x$ function.

8. Let $H : \mathbb{R} - \{1\} \to \mathbb{R} - \{1\}$ ve defined by the formula

$$H(x) \quad = \quad \frac{x+1}{x-1} \; .$$

Verify that H is 1–1 onto, and that it is equal to its own inverse.

9. Fin the inverse function to $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x + 2$.

10. Let

$$f(x) \quad = \quad \frac{1}{3 + \cos^4 x} \; .$$

Find an interval $A = [a, b]$ in the real line such that $f|A$ is 1–1, and find an interval $B = [c, d]$ in the real line such that B is contained in $f[A]$.

11. Let $f : X \to Y$ be a function, let $A \subset X$, and let $B \subset Y$. Prove that

$$f[A] \quad = \quad f\left[f^{-1}\left[f[A]\right]\right] \qquad \text{and} \qquad f^{-1}[B] \quad = \quad f^{-1}\left[f\left[f^{-1}[B]\right]\right] \; .$$

12. Given two functions $f, g : \mathbb{R} \to \mathbb{R}$, define their sum $f + g$ as above. If $h : \mathbb{R} \to \mathbb{R}$ is an arbitrary function,prove that $(f + g) \circ h = f \circ h + g \circ h$. Also, give a counterexample to show that $f \circ (g + h)$ need not be equal to $f \circ g + f \circ h$.

13. Given two functions $f, g : \mathbb{R} \to \mathbb{R}$, suppose also that $f \circ g = g \circ f$. Does it follow that $f = g$? Give a proof if the answer is yes and a counterexample if the answer is no.

14. Given two functions $f, g : \mathbb{R} \to \mathbb{R}$ and a constant C, for $h = f, g$ define the function $C \cdot h$ by so that its value at x is $C \cdot f(x)$. Prove that $K \cdot (f \circ g) = (C \cdot f) \circ g$, and give a counterexample to show that $f \circ (Cg)$ is not necessarily equal to the first two functions.